Electoral System Choice

THOMAS BRAMBOR
New York University

WILLIAM ROBERTS CLARK
New York University

MATT GOLDER
New York University

November 14, 2004

1   Introduction

In 'Understanding Interaction Models: Improving Empirical Analyses', we report the results of several replications that we conducted of analyses examining electoral institutions and party systems. One of these replications was of Carles Boix's article 'Setting the Rules of the Game: The Choice of Electoral Systems in Advanced Democracies' which appeared in the *British Journal of Political Science* in 1999. In our article, we said that:

> 'In an award-winning article in the *American Political Science Review*, Boix (1999) examines the factors that determine electoral system choice in advanced democracies. He makes two main conclusions. First, ethnic or religious fragmentation encourages the adoption of proportional representation in small and medium-sized countries (621). He draws this conclusion based on a model that includes an interaction term between ethno-religious fragmentation and country size. However, he does not include either of the constitutive terms. When these terms are included, there is no longer any evidence that ethno-religious fragmentation ever affects the adoption of proportional representation. The second conclusion is that countries are more likely to shift to proportional representation when the proportion of Socialist votes and the effective number of non-socialist parties are both large. This conclusion comes from a model in which there is an interaction term between the strength of socialist parties and the number of non-socialist

parties but no constitutive terms. In this case, the coefficient on the interaction term from the primary model remains significant (albeit only at the 90% level now) once the constitutive terms are included, but its magnitude increases by 340%. Thus, the original analysis considerably underestimates the interactive effect of these two variables. Moreover, the failure to include constitutive terms means that the predicted electoral thresholds reported by Boix are off by up to 80%.'

The replication that justifies the description given above is described below. At this point, we would like to thank Carles Boix for providing his data.

2 Replication

In his article, Boix examines the factors that determine the electoral threshold in advanced industrial democracies during the interwar period. In an appendix, he also investigates the determinants of electoral thresholds for democracies that emerged after 1945.

2.1 Model

Boix does not explicitly state his model in the article. However, the specification for Model 1 in Table 1 of his article is:

$$
\begin{aligned}
EffectiveThreshold = \beta_0 \quad &+ \quad \beta_1 Threat + \beta_2 Area + \beta_3 TradeOpenness \\
&+ \quad \beta_4 Population + \beta_5 Fragmentation + \epsilon
\end{aligned} \tag{1}
$$

where *EffectiveThreshold* is the average effective threshold, *TradeOpenness* is a measure of trade openness calculated as the log value of the sum of exports and imports as a proportion of GDP during the first years of the interwar period and at the time the new democratic regimes were established after 1945, *Population* is the log of population size, *Area* is the log value of the geographic area in thousands of km^2, and *Fragmentation* is measured by ethnic and linguistic fractionalization. *Threat* is actually an interaction term between *Socialism* and *OldParties*, where *Socialism* is the percentage of votes won by socialist parties and *OldParties* is the effective number of non-Socialist

parties.[1] Thus, the results from Model 1 in Table 1 of his article actually comes from the following model:

$$
\begin{aligned}
EffectiveThreshold = \beta_0 \quad & + \quad \beta_1 Socialism * OldParties + \beta_2 Area + \beta_3 TradeOpenness \\
& + \quad \beta_4 Population + \beta_5 Fragmentation + \epsilon
\end{aligned}
\tag{2}
$$

As we note in our article, 'Understanding Interaction Models', scholars employing interaction models should include all of the constitutive terms. However, it is easy to see that Boix omits two constitutive terms in his specification shown above. These are *Socialism* and *OldParties*. The correct and fully-specified model is shown below:

$$
\begin{aligned}
EffectiveThreshold = \gamma_0 \quad & + \quad \gamma_1 Socialism + \gamma_2 OldParties \\
& + \quad \gamma_3 Socialism * OldParties \\
& + \quad \gamma_4 Area + \gamma_5 TradeOpenness \\
& + \quad \gamma_6 Population + \gamma_7 Fragmentation + \epsilon
\end{aligned}
\tag{3}
$$

We should note that Boix does actually estimate a model with both constitutive terms included (Model 2). However, he does not include all of the other independent variables shown above. Moreover, the predictions that he makes are based on the under-specified model shown earlier in (2).

2.2 Hypotheses

Boix tests several hypotheses in his article. However, we are only interested in his claims about the effect of (i) ethnic fragmentation and (ii) the strength of socialist parties and the number of non-socialist parties on the electoral threshold. These effects are the central focus of his article. Boix claims that ethnic fragmentation encourages the adoption of proportional representation (lower electoral threshold) in small and medium-sized countries (621). He also claims that countries are more likely to shift to proportional representation when the proportion of Socialist votes and the effective number of old non-socialist parties are both large. This last claim is somewhat ambiguous. We are not sure whether he means that the marginal effect of a strong socialist party will decrease

---

[1]Although the article states that *Threat* is calculated as *Socialism*OldParties*, this is not exactly accurate. We noticed that *Threat* is actually calculated as $\frac{Socialism*OldParties}{100}$ in the dataset.

the electoral threshold when there are many old, non-socialist parties *or* that the marginal effect of more old, non-socialist parties decreases the electoral threshold when there are strong socialist parties.[2] In the replication outlined below, we examine both ways of evaluating Boix's second claim.

2.3 Replication for Interwar Period

Although Boix does examine the choice of electoral threshold in democracies that emerged after 1945, the central focus of the article is the choice of electoral threshold in the interwar period. As a result, we focus on this choice first. The results from the replication are shown in Table 1 on the next page. Boix estimates three different models. Models 1, 2 and 3 in Table 1 refer to Models 1, 2 and 3 on page 619 of Boix's article. The first column for each model reports the coefficients from the article (reported), the second column reports the coefficients that we managed to replicate using the original model specification (replication), and the third column shows the results from a fully-specified model (full). Model 1 seems to be Boix's preferred specification since this is the model used to calculate his predictions in Table 2 of his article.

2.3.1 Replication Issues

As the reader can see, we were able to replicate the results from Model 1 and Model 2 exactly. The only difference between our results and those of Boix in Models 1 and 3 is that Boix reports that the coefficient on *Area* is significant at the 95% level for Model 1 when in fact it is only significant at the 90% level. We were unable to replicate the results from Model 3 exactly. In the dataset provided by Boix, there was no variable for the 'area dummy' and so we were forced to create this ourselves. Supposedly, this dummy is 0 for countries bigger than 450,000km^2, 1 otherwise. So we created a dummy=0 if *Area*$> 450,000 km^2$, 1 otherwise. Although we were unable to replicate the results from Model 3 in the article exactly, we did come extremely close.

---

[2] Boix might mean that the marginal effect of increasing the strength of socialism AND the number of old, non-socialist parties reduces the electoral threshold. This marginal effect is, in fact, given by the coefficient on the interaction term i.e. $\frac{\partial^2 EffectiveThreshold}{\partial Socialism\ \partial OldParties} = \gamma_3$. However, Boix does not make this argument explicitly. Moreover, he gives no evidence that it makes sense to think of both *Socialism* and *OldParties* increasing at the same time. On the whole, we believe that Boix is interested in how the effect of stronger socialist parties on the electoral threshold depended on the number of old, non-socialist parties.

Table 1: The Choice of Electoral Rules in the Interwar Period

Regressor	Model 1 reported	Model 1 replication	Model 1 full	Model 2 reported	Model 2 replication	Model 2 full	Model 3 reported	Model 3 replication	Model 3 full
Threat	-11.58**	-11.58**	-39.62*	-40.17**	-40.17**	-40.17**	-13.55**	-13.34**	-49.95**
	(4.39)	(4.39)	(21.95)	(18.92)	(18.92)	(18.92)	(4.94)	(4.92)	(22.71)
Strength of Socialism			0.74	78.96	0.79	0.79			1.12
			(0.72)	(58.71)	(0.59)	(0.59)			(0.72)
Effective Number Old Parties			8.96	8.92	8.92	8.92			8.65
			(5.74)	(5.18)	(5.18)	(5.18)			(6.08)
Geographical Area	10.19**	10.19*	10.35*	9.67**	9.67**	9.67**			
	(5.24)	(5.24)	(5.16)	(2.17)	(2.17)	(2.17)			
Trade Openness	1.15	1.15	0.27						
	(8.36)	(8.36)	(8.26)						
Population	0.04	0.04	-0.18						
	(1.54)	(1.54)	(1.94)						
Ethnic Fragmentation	-4.32	-4.32	-6.51						-5.59
	(13.34)	(13.34)	(13.72)						(26.27)
Fragmentation*AreaDummy							-32.14**	-33.17**	-7.13
							(14.61)	(14.74)	(32.03)
AreaDummy									-17.03
									(11.45)
Constant	-2.48	-2.48	-22.88	-22.74	-22.74	-22.74	31.21**	31.30**	23.68
	(43.12)	(43.12)	(46.68)	(17.14)	(17.14)	(17.14)	(4.70)	(4.68)	(26.13)
Observations	22	22	314	22	22	22	22	22	22
R^2	0.61	0.61	-443.76	0.67	0.67	0.67	0.39	0.40	0.60

* $p < 0.10$; ** $p < 0.05$ (two-tailed); Standard errors in parentheses

5

2.4 Interpretation for Interwar Period

2.4.1 The Effect of *Ethnic Fragmentation*

Boix finds that ethnic fragmentation has no significant effect on the electoral threshold in Model 1. He goes on to claim that this is because he did not take account of the modifying effect of country size. He believes that ethnic fragmentation should only lower the electoral threshold (encourage PR) in small countries i.e. AreaDummy=1. It should have no effect in large countries. Boix tests this conditional hypothesis in Model 3 by including an interaction term between *Ethnic Fragmentation* and *AreaDummy*. He claims support for his hypothesis because the coefficient on the interaction term is negative and significant. However, as we state in our article 'Understanding Interaction Models' analysts should include all of the constitutive terms. Boix does not do this. In fact, he does not include either of the constitutive terms - he omits *Ethnic Fragmentation* and *AreaDummy* as separate variables. This means that his results are potentially biased.

So, what happens when we include all of the constitutive terms in Model 3? Now, none of the coefficients on *Ethnic Fragmentation*, *AreaDummy*, or the interaction term are significant. However, all we can infer from this is that ethnic heterogeneity has no significant effect on electoral thresholds in large countries - the marginal effect is -5.59 (26.27). Standard errors are shown in parentheses. But what about in small countries? The marginal effect of ethnic fragmentation in small countries is -12.71 (16.93). In other words, it does not matter whether the country is small or bid, ethnic fragmentation has no significant effect on electoral thresholds. This is direct contradiction to the claim made by Boix.

2.4.2 The Effect of *Socialism* and *Old Parties*

The coefficient on *Threat* in Model 1 is negative and significant. From this Boix infers support for his claim that electoral thresholds will be lower when socialist parties are strong and the number of old, non-socialist parties is high. However, Boix again fails to include any of the constitutive terms i.e. *Socialism* and *Old Parties* are omitted. This means that his results in Model 1 are potentially biased. Are they, in fact, biased? The inclusion of the constitutive terms clearly shows that they are. The fully-specified version of Model 1 shows that the coefficient on the interaction term *Threat* increases by about 350% compared to the under-specified model (In Model 3, the same

coefficient increases by 370%). Thus, Boix strongly underestimates the interactive relationship between *OldParties* and *Socialism* in Models 1 and 3.

As we said earlier, Boix does estimate a fully-specified model in Model 2 (although this model does not include all of the same independent variables as either Model 1 and Model 3). In fact, the results from this model clearly show that the coefficient on the interaction term is severely underestimated in Models 1 and 3 as we just showed. However, Boix mistakenly claims that this increase in the coefficient on the interaction term in Model 2 is due to multicollinearity. This leads him to revert back to Model 1 when drawing his inferences. Presumably Boix thinks that the large change in the coefficient on the interaction term is a sign of multicollinearity as it might be in a linear-additive model. However, as we note in our article, 'Understanding Interaction Models', this is a common mistake among analysts employing interaction models. The reason why the coefficient changed is not because of multicollinearity but because the model is now conditional due to the inclusion of constitutive terms.[3] Thus, although Boix does estimate a correct and fully-specified model, he does not realize that this is the case.

Moreover, he incorrectly interprets the coefficients on the constitutive terms in Model 2. He says that they have the wrong sign and therefore do not support his claims that the effect of *Socialism* and *OldParties* should reduce the electoral threshold.[4] This is clear evidence that he is interpreting these coefficients as if they were unconditional marginal effects. He is ignoring the fact that they only indicate the effect of *Socialism* when *OldParties*=0 and the effect of *OldParties* when *Socialism*=0. Thus, they neither confirm or reject his unconditional hypotheses that socialism and the number of non-socialist parties should reduce the electoral threshold. As we note in our article on interaction models, it is not possible to determine the average or unconditional effects of variables in a multiplicative interaction model without some knowledge of the distribution of the modifying variables. This seems to be what Boix is trying to do when interpreting the coefficients on the constitutive terms in Model 2, though.

---

[3]By just including *Threat* in Model 1, the computer has no way of knowing that *Threat* is really an interaction term. As a result, the computer treats it as if it were a normal variable. Thus, Model 1 is equivalent to a linear additive model for the computer. It only becomes an interactive model when we add the constitutive terms in Model 2.

[4]Boix did not only claim that electoral thresholds should be lower when socialist parties were strong AND the number of non-socialist parties was high. He also said that strong socialist parties should lead to lower thresholds (unconditional claim) and that more non-socialist parties should lower thresholds (unconditional claim).

2.4.3 Simulation

Boix does not simply present the results from his regressions. He goes further and tries to give his results more substantive meaning through the use of simulations. His simulated results are shown in Table 2 of his article. He looks at the predicted electoral threshold when *Area*=3 and *TradeOpenness*, *Population*, and *Fragmentation* are all set to their mean values. He uses the results from the under-specified Model 1 in Table 1. We were able to replicate his results relatively accurately - they will not be identical because of the use of simulation. We then compared these predictions with the predicted electoral threshold from a simulation using the results from the correct and fully-specified Model 1. The predicted values for electoral threshold from (i) the original analysis, (ii) our replication, and (iii) the fully-specified model are shown in Table 2. Thus, Table 2 allows us to compare the predicted electoral thresholds from the under-specified model and the fully-specified model.

Table 2: Predicted Effective Electoral Threshold in the Interwar Period

	Boix's Original Analysis Percentage of Socialist Votes			
# Old Parties	10	20	30	40
2	29.2	26.9	24.6	22.3
	(3.7)	(3.3)	(3.2)	(3.2)
4	26.9	22.3	17.6	13.0
	(3.3)	(3.2)	(4.0)	(5.3)
6	24.6	17.6	10.7	3.8
	(3.2)	(4.0)	(6.0)	(8.3)

	Replication			
# Old Parties	10	20	30	40
2	29.27	26.98	24.60	22.29
	(4.21)	(4.04)	(3.81)	(3.97)
4	26.72	22.44	17.30	12.69
	(3.90)	(3.88)	(4.74)	(5.80)
6	24.56	17.42	10.53	3.47
	(3.92)	(4.67)	(6.64)	(8.63)

	Fully Specified			
# Old Parties	10	20	30	40
2	24.96	23.82	23.64	22.96
	(5.66)	(4.23)	(4.95)	(7.25)
4	34.74	26.15	17.89	9.04
	(6.12)	(4.87)	(4.74)	(5.94)
6	44.22	28.49	11.73	-4.03
	(12.27)	(8.44)	(8.08)	(12.36)

(Standard errors are given in parentheses)

It is fairly easy to see that there are distinct differences in the predicted thresholds between these two models. In some cases, the fully-specified model predicts an electoral threshold fully 80% larger than Boix's model. Thus, the under-specified models are quite far off in terms of their predictions.
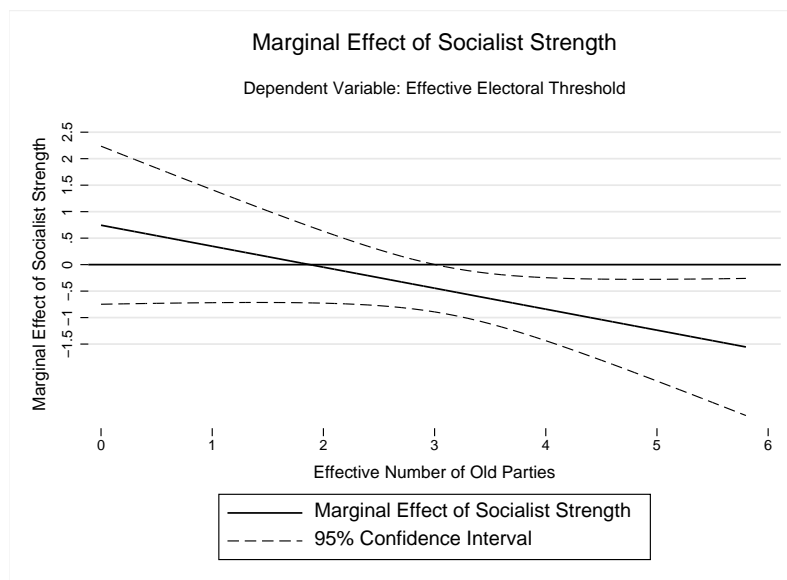
2.4.4   Marginal Effects

Of course, we are not primarily interested in predicted effective thresholds at all. We want to know how *Socialism* and *Old Parties* affect the level of effective thresholds. To see this we need to calculate how the marginal effect of *Socialism* and *OldParties* varies across their respective modifying variables. The marginal effect *Socialism* is

$$\frac{\partial EffectiveThreshold}{\partial Socialism} = \gamma_1 + \gamma_3 OldParties \tag{4}$$

We plot the marginal effect of *Socialism* across the observed range for the number of old, non-socialist parties in Figure 1. This figure indicates that an increase in *Socialism* does decrease the electoral threshold once the effective number of non-socialist parties is greater than 3.1. This is the case for 10 out of 22 observations in the sample.

Figure 1: Marginal Effect of Socialist Strength on the Electoral Threshold in the Interwar Period.

The marginal effect Old Parties is

$$\frac{\partial EffectiveThreshold}{\partial OldParties} = \gamma_2 + \gamma_3 Socialism \tag{5}$$

We plot the marginal effect of *OldParties* across the observed range for the strength of socialist parties in Figure 2. It is fairly easy to see that Figure 2 indicates that an increase in the number of non-socialist parties never has any effect on the electoral threshold even when the strength of socialist parties is high.

Figure 2: Marginal Effect of the Number of Non-Socialist Parties on the Electoral Threshold in the Interwar Period
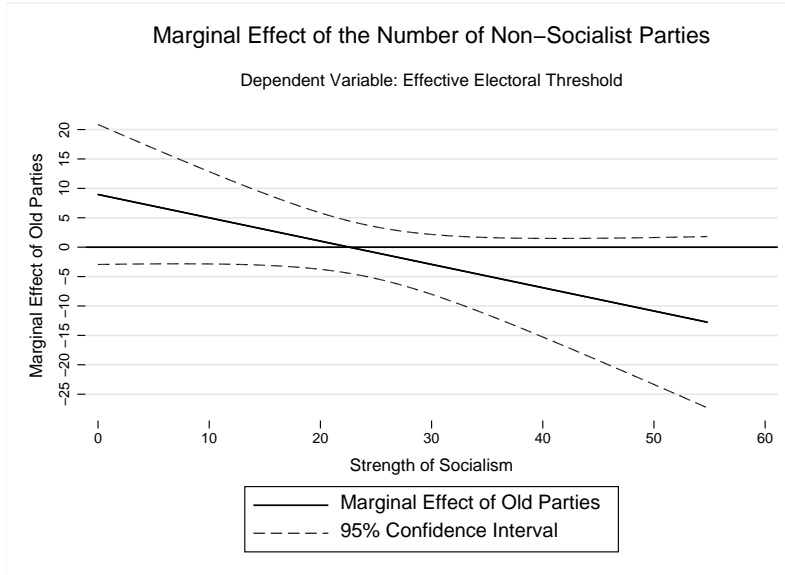
Thus, Figure 1 supports Boix's hypothesis regarding the effect of *Socialism* on the electoral threshold. However, Figure 2 does not support his argument regarding the effect of the number of non-socialist parties.

2.5   Interpretation for Interwar Period and New Democracies after 1945

As we have already stated, the central focus of Boix's article is the choice of electoral thresholds in the interwar period. However, he presents additional results in Appendix B of his article where he also adds new democracies that emerged after 1945 to those in the interwar period. Boix claims that the results hardly vary from the sample that only included interwar democracies.

2.5.1 Results

We were able to replicate Models 1 and 2 exactly in Appendix B exactly. We were unable to replicate the results from Model 3 perfectly, although we got very close again. The results of our replication are shown in Table 3 on the next page.

2.5.2 Interpretation

As before, we find that Models 1 and 3 that omit the constitutive terms *Socialism* and *OldParties* severely underestimate the coefficient on the interaction term by about 300%. If we look at Model 3, we can again see that there is no evidence that the effect of ethnic fragmentation on electoral threshold depends on the size of the country once the model is fully-specified. The coefficient on the interaction term in Model 3 is far from significant and is, in fact, the wrong sign. The marginal effect of ethnic fragmentation in large countries is -9.08 (21.43). Standard errors are again in parentheses. The marginal effect of ethnic fragmentation in small countries is -6.65 (13.51). In other words, there is no evidence that ethnic fragmentation ever affects the electoral threshold irrespective of the size of the country.

2.5.3 Marginal Effects

As before, we are primarily interested in the marginal effects of the variables and not the model parameters *per se*. In Figures 3 and 4 we plot the marginal effect of *Socialism* across the observed range of *OldParties* and the marginal effect of *OldParties* across the observed range of *Socialism* respectively. Figure 3 shows that an increase in *Socialism* does decrease the electoral threshold so long as the number of non-Socialist parties is greater than 2.5. 19 out of 31 observations have more non-Socialist parties than this. Figure 4 shows that an increase in *OldParties* does decrease the electoral threshold so long as socialist parties win more than 36.6% of the vote. Only 6 observations out of 31 have socialist parties winning more votes than this. Note that when we just looked at the interwar period, we found no evidence that an increase in the number of non-socialist parties ever had any effect on the electoral threshold. Thus, although Boix argues that the results from the interwar and post-war sample hardly vary from the interwar sample, this is not exactly the case.

Table 3: The Choice of Electoral Rules in the Interwar and Post-1945 Period

Regressor	Model 1 reported	Model 1 replication	Model 1 full	Model 2 reported	Model 2 replication	Model 2 full	Model 3 reported	Model 3 replication	Model 3 full
Threat	-11.27**	-11.27**	-32.95*	-34.07**	-34.07**	-34.07**	-13.24**	-13.12**	-39.86**
	(3.39)	(3.39)	(16.90)	(15.82)	(15.82)	(15.82)	(3.89)	(3.87)	(17.55)
Strength of Socialism			0.54	57.70	0.58	0.58			0.68
			(0.50)	(45.86)	(0.46)	(0.46)			(0.50)
Effective Number Old Parties			7.20	7.31	7.31	7.31			6.84
			(4.76)	(4.50)	(4.50)	(4.50)			(4.89)
Geographical Area	11.33**	11.33**	11.52**	9.89**	9.89**	9.89**			
	(3.45)	(3.45)	(3.42)	(1.99)	(1.99)	(1.99)			
Trade Openness	2.96	2.96	1.93						
	(4.43)	(4.43)	(4.56)						
Population	0.08	0.08	-0.31						
	(1.32)	(1.32)	(1.50)						
Ethnic Fragmentation	-2.26	-2.26	-4.62						
	(11.18)	(11.18)	(11.66)						
Fragmentation*AreaDummy							-31.45**	-32.18**	2.43
							(12.50)	(12.58)	(25.44)
AreaDummy									-18.05
									(8.61)
Constant	-13.03	-13.03	-26.99	-18.13	-18.13	-18.13	30.02**	30.09**	23.54
	(22.51)	(22.51)	(27.12)	(14.40)	(14.40)	(14.40)	(4.70)	(3.95)	(16.25)
Observations	31	31	31	31	31	31	31	31	31
R^2	0.60	0.60	0.64	0.42	0.42	0.42	0.37	0.38	0.60

* $p < 0.10$; ** $p < 0.05$ (two-tailed); Standard errors in parentheses

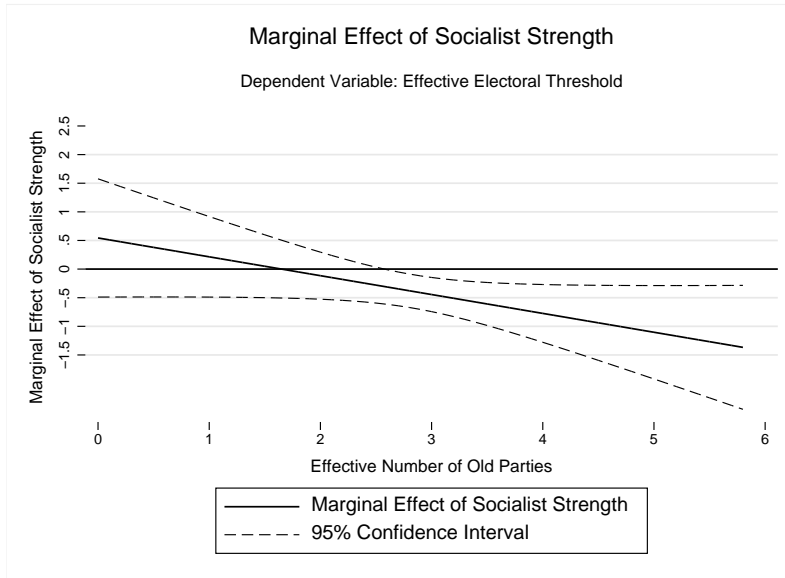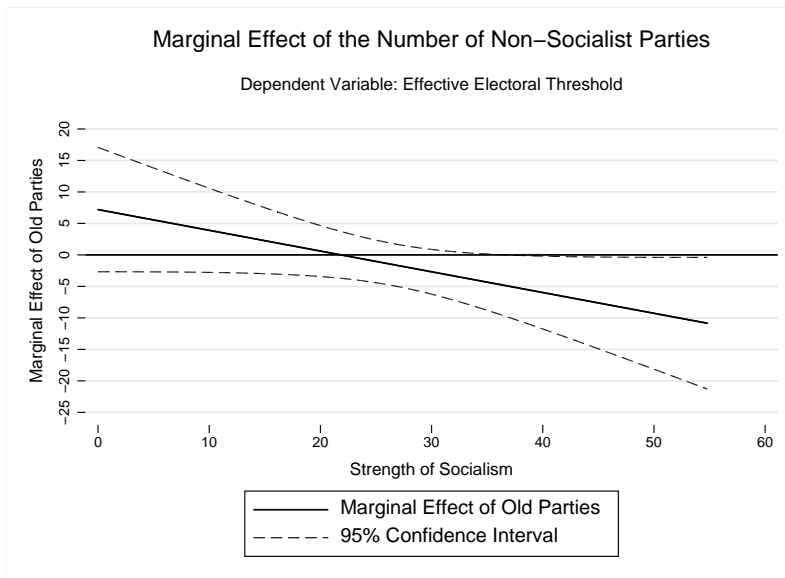Figure 3: Marginal Effect of Socialist Strength on the Electoral Threshold

Figure 4: Marginal Effect of the Number of Non-Socialist Parties on the Electoral Threshold

3   Conclusion

Our replication shows that Boix is wrong to claim that ethnic or religious fragmentation encourages the adoption of proportional representation in small and medium-sized countries. There is no evidence for this once his model is fully-specified. Ethnic fragmentation never has a significant effect on the electoral threshold. Boix is correct in saying that there is an interactive effect between *Socialism* and *OldParties*. However, he severely underestimates this effect by not including constitutive terms. Not only this, but Boix's preferred model (Model 1) is not really an interaction model because the constitutive terms are not included. As a result, the *Threat* variable is treated (and really is) a single variable. When we examine the marginal effect of socialist parties we find strong evidence in both samples (interwar and postwar) that it decreases the electoral threshold once the number of non-socialist parties is sufficiently large. This is exactly as one would expect. When we examine the marginal effect of non-socialist parties we find that it never has a significant effect on the electoral threshold in the sample that only includes interwar democracies. However, it does have a significant reductive effect on electoral thresholds once the votes for socialist parties are sufficiently large if we examine democracies from the interwar period along with democracies that emerged after 1945. Thus, the evidence concerning the marginal effect of non-socialist parties is mixed.

4   STATA Code

The STATA do-file to replicate these results is called boix.do

References

Boix, Carles. 1999. "Setting the Rules of the Game: The Choice of Electoral Systems in Advanced Democracies." *American Political Science Review* 93:609–624.