Problems with Group Decision Making
There are two ways of evaluating political systems:

1. **Consequentialist ethics** evaluate actions, policies, or institutions in regard to the outcomes they produce.

2. **Deontological ethics** evaluate actions, policies, or institutions in light of the rights, duties, or obligations of the individuals involved.
Many people like democracy because they believe it to be a fair way to make decisions.

One commonsense notion of fairness is that group decisions should reflect the preferences of the majority of group members.

Most people probably agree that a fair way to decide between two options is to choose the option that is preferred by the most people.

At its heart, democracy is a system in which the majority rules.
An actor is **rational** if she possesses a **complete** and **transitive** preference ordering over a set of outcomes.
An actor has a complete preference ordering if she can compare each pair of elements (call them $x$ and $y$) in a set of outcomes in one of the following ways - either the actor prefers $x$ to $y$, $y$ to $x$, or she is indifferent between them.

An actor has a transitive preference ordering if for any $x$, $y$, and $z$ in the set of outcomes, it is the case that if $x$ is weakly preferred to $y$, and $y$ is weakly preferred to $z$, then it must be the case that $x$ is weakly preferred to $z$. 
Condorcet’s paradox illustrates that a group composed of individuals with rational preferences does not necessarily have rational preferences as a collectivity.

Individual rationality is not sufficient to ensure group rationality.
Imagine a city council made up of three individuals that must decide whether to:

1. Increase social services (I)
2. Decrease social services (D)
3. Maintain current levels of services (C)
Let's assume that the council employs majority rule to make its group decisions. In this particular example, this means that any policy alternative that enjoys the support of two or more councillors will be adopted. How should the councillors vote, though? It's not obvious how they should vote given that there are more than two alternatives. One way they might proceed is to hold a round-robin tournament that pits each alternative against every other alternative in a set of "pair-wise votes" — \( I \) versus \( D \), \( I \) versus \( C \), and \( C \) versus \( D \) — and designates as the winner whichever alternative wins the most contests. If we assume that the councillors all vote for their most preferred alternative in each pair-wise contest (or round), then we see that \( D \) defeats \( I \), \( I \) defeats \( C \), and \( C \) defeats \( D \). The outcomes of these pair-wise contests and the majorities that produce them are summarized in Table 11.2. Notice that there is no alternative that wins most often — each alternative wins exactly one pair-wise contest. This multiplicity of "winners" does not provide the council with a clear policy direction. In other words, the council fails to reach a decision on whether to increase, decrease, or maintain current levels of social service provision.

This simple example produces several interesting results that we now examine in more detail. The first is that a group of three rational actors (the councillors) make up a group (the council) that appears to be incapable of making a rational decision for the group as a whole. What do we mean by "rational"? When political scientists use the word rational, they have a very specific meaning in mind. An actor is said to be rational if she possesses a complete and consistent set of preferences.

### Table 11.1

<table>
<thead>
<tr>
<th>Left-wing councillors</th>
<th>Centrist councillors</th>
<th>Right-wing councillors</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I &gt; C &gt; D )</td>
<td>( C &gt; D &gt; I )</td>
<td>( D &gt; I &gt; C )</td>
</tr>
</tbody>
</table>

Note: \( I \) = increased social service provision; \( D \) = decreased social service provision; \( C \) = maintenance of current levels of social service provision; \( > \) = "is strictly preferred to."
Let’s suppose that the council employs majority rule to make its group decision.

One possibility is a round-robin tournament.

A round-robin tournament pits each competing alternative against every other alternative an equal number of times in a series of pair-wise votes.
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A group of rational individuals is incapable of making a rational decision for the group as a whole.

There is no ‘majority’ to speak of – a different majority supports the winning alternative or outcome in each round.

<table>
<thead>
<tr>
<th>Round</th>
<th>Contest</th>
<th>Winner</th>
<th>Majority that produced victory</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Increase vs. decrease</td>
<td>$D$</td>
<td>Centrist and right</td>
</tr>
<tr>
<td>2</td>
<td>Current vs. increase</td>
<td>$I$</td>
<td>Left and right</td>
</tr>
<tr>
<td>3</td>
<td>Current vs. decrease</td>
<td>$C$</td>
<td>Left and centrist</td>
</tr>
</tbody>
</table>
To see why, imagine that maintaining current spending on social services is the status quo and ask yourself who would benefit from a change. The answer is that both the left- and right-wing councillors would like to propose a change. The right-wing council member prefers a decrease in social service provision to the status quo. If he proposed a decrease, however, both the centrist and left-wing councillors would vote against the proposal. Similarly, the left-wing council member prefers an increase in social service provision to the status quo. But if he proposed an increase, both the centrist and right-wing councillors would vote against the proposal. In other words, with this new profile of preferences in the group, there is no cycle of majorities, and as a result, current levels of spending constitute a stable outcome. In effect, the group now behaves as if it were an individual with transitive (and complete) preferences—it prefers current levels of social service provision to a decrease and a decrease to an increase.

The point here is that majority rule is not necessarily incompatible with rational group preferences. All that Condorcet showed was that it is possible for a group of individuals with transitive preferences to produce a group that behaves as if it has intransitive preferences. As a result, Condorcet's paradox erodes our confidence in the ability of majority rule to produce
Our example demonstrates how a set of rational individuals can form a group with intransitive preferences.

In the real world, though, we see deliberative bodies make decisions all the time and they do not appear to be stuck in an endless cycle.

Why?
There are two broad reasons for this:

1. Preference orderings.

2. Decision-making rules.
The councillors having a particular set of preference orderings.

Suppose the right-wing councillor’s preferences are now a mirror image of the left-wing councillor’s.

His preferences are now $D > C > I$ instead of $D > I > C$. 
If the right-wing councillor’s preferences are $D > C > I$, then $C$ is a **Condorcet winner**.

An option is a **Condorcet winner** if it beats all of the other options in a series of pair-wise contests.
Majority rule is not necessarily incompatible with rational group preferences.

Condorcet’s Paradox only shows that it is possible for a group of individuals with transitive preferences to produce a group that behaves as if it has intransitive preferences.
How often are individuals likely to hold preferences that cause intransitivity?
stable outcomes only to the extent that we expect actors to hold the preferences that cause group intransitivity. So how likely is it that transitive individual preferences will lead to group intransitivity? Modern scholars have analyzed this problem in detail and found, assuming that all preference orderings are equally likely, that the likelihood of group intransitivity increases with the number of alternatives under consideration or the number of voters or both. In Table 11.3, we show estimates of the share of all possible strict preference orderings that fail to produce a Condorcet winner (that is, that produce group intransitivity) as the numbers of voters and alternatives increase (Riker 1982, 122).

As Table 11.3 illustrates, the example of the city council that we started with, in which a Condorcet winner fails to emerge from a contest among three alternatives and three voters, is indeed a rarity. Nearly all (94.4 percent) of the logically possible strict preference orderings produce a Condorcet winner and, hence, a stable outcome. As the number of voters increases, however, the probability of group intransitivity rises to some limit. When the number of alternatives is relatively small, this limit is still small enough that most of the logically possible preference orderings will not lead to group intransitivity. In contrast, although an increase in the number of alternatives also increases the probability of group intransitivity, this process continues until the point at which group intransitivity is certain to occur. In other words, as the number of alternatives goes to infinity, the probability of group intransitivity converges to one—even when the number of voters is small. This is an extremely important result because many political decisions involve a choice from, essentially, an infinite number of alternatives.

### Table 11.3: Proportion of Possible Strict Preference Orderings without a Condorcet Winner

<table>
<thead>
<tr>
<th>Number of alternatives</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.056</td>
<td>0.069</td>
<td>0.075</td>
<td>0.078</td>
<td>0.080</td>
<td>0.088</td>
</tr>
<tr>
<td>4</td>
<td>0.111</td>
<td>0.139</td>
<td>0.150</td>
<td>0.156</td>
<td>0.160</td>
<td>0.176</td>
</tr>
<tr>
<td>5</td>
<td>0.160</td>
<td>0.200</td>
<td>0.215</td>
<td></td>
<td></td>
<td>0.251</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>0.202</td>
<td></td>
<td></td>
<td></td>
<td>0.315</td>
</tr>
<tr>
<td>Limit</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
In general, we cannot rely on majority rule to produce a coherent sense of what the group wants, especially if there are no institutional mechanisms for keeping the number of voters small or weeding out some of the alternatives.
Many political decisions involve bargaining and hence an infinite number of alternatives!
Condorcet’s Paradox indicates that restricting group decision making to sets of rational individuals is no guarantee that the group as a whole will exhibit rational tendencies.

Group intransitivity is unlikely when the set of feasible options is small, but it is almost certain when the set of feasible alternatives gets large.

As a result, it is impossible to say that the majority ‘decides’ except in very restricted circumstances.
The analytical insight from Condorcet’s Paradox suggests that group intransitivity should be common.

But we observe a surprising amount of stability in group decision making in the real world.
Perhaps this has something to do with the decision-making rules that we use.

1. The Borda count.

2. A powerful agenda setter.
The **Borda count** asks individuals to rank potential alternatives from their most to least preferred and then assign points to reflect this ranking.

The alternative with the most ‘points’ wins.
A 2 to his second-best option, and a 1 to his least preferred option. The weighted votes for each alternative are then summed, and the alternative with the largest score wins. Using the same preferences as shown earlier in Table 11.1, the Borda count would again be indecisive in determining whether to increase, decrease, or maintain current levels of social service provision. This is because each alternative would garner a score of 6. This is shown in Table 11.4.

Although the indecisiveness of the Borda count is once again an artifact of the particular preference ordering we are examining, a more troubling aspect of this decision rule can be seen if we consider the introduction of a possible fourth alternative. Let’s assume, for example, that the councillors consider a new alternative: maintain current spending levels for another year (perhaps it’s an election year) but commit future governments to a decrease in spending of, say, 10 percent in each successive year. Suppose that the left-wing councillor likes this new option the least, the right-wing councillor prefers it to all alternatives except an immediate decrease, and the centrist councillor prefers all options except an increase to this new alternative. The preference ordering for each of the council members over the four alternatives is summarized in Table 11.5.

If we apply the Borda count in this new situation by assigning a 3 to each councillor’s most preferred alternative, a 2 to his second-best alternative, a 1 to his third-best alternative, and a 0 to his least preferred alternative, then we find that the vote tally looks like the one shown in Table 11.6. As you can see, the council now has a strict preference ordering over the alternatives. Based on the councillors’ votes, the council would decrease the level of social service provision.

6. We could, of course, conclude that the group actually is indifferent between these alternatives, given this aggregation of citizen preferences. Doing so, however, requires us to make what political scientists call “interpersonal comparisons of utility.” For example, we would have to believe that the welfare improvement that a left-wing councillor feels when a decrease in social service provision is replaced by an increase is exactly equal to the sum of the decline in welfare experienced by the centrist and left-wing councillors when this happens. Most modern scholars are reluctant to make these types of interpersonal comparisons of utility and so would be reluctant to make normative statements about the appropriateness of this outcome.

7. This example is not as fanciful as it might sound. In fact, it shares many qualities with the “balanced budget” proposals of politicians who are all too eager to be “fiscally conservative” tomorrow (when an election is no longer looming).

### Table 11.4

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Left-wing</th>
<th>Centrist</th>
<th>Right-wing</th>
<th>Borda count total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase spending</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Decrease spending</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Current spending</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Using the same preferences as before, the Borda count does not provide a clear winner either.
A more troubling aspect of this decision rule can be seen if we consider the introduction of a fourth alternative, future cuts ($FC$).

You will immediately notice that something very strange has happened. Despite the fact that the new alternative receives a lower score than all of the original options and that it is not the first choice of any of the councillors, its addition as an active alternative for consideration changes how the councillors, as a collectivity, rank the three original options. In doing so, it changes the outcome of the vote. Whereas the group had previously been "indifferent" between the three original options, it now possesses a strict and transitive preference ordering over them, with "decreased spending" as the group's "most preferred" outcome. Note that this is the case despite the fact that none of the councillors has changed the way that he rank orders $I$, $D$, and $C$. In effect, the choice that the council now makes has been influenced by the introduction of what might be called an "irrelevant alternative." As this example illustrates, the Borda count does not demonstrate the property that political scientists refer to as "independence from irrelevant alternatives."  

8. Technically, the "independence from irrelevant alternatives" (IIA) property in the social choice literature refers to the independence from the "ranking" (and not the "presence") of an irrelevant alternative. This is the requirement that the ranking of an irrelevant alternative in a fixed set of alternatives should not affect the alternative that is chosen (Arrow 1963; Sen 1970). Our city council example can be understood in these terms too. For example, we can imagine that the city councillors all originally ranked the alternative of future spending cuts last but through some kind of deliberation process came to rank it in the way shown in Table 11.5. When the future spending cuts are ranked last, the council is indifferent between $D$, $I$, and $C$. But when the future spending cuts are ranked according to the preference orderings in Table 11.5, then the council has a strict preference ordering, $D > C > I$.

### Table 11.5 City Council Preferences for the Level of Social Service Provision (Four Alternatives)

<table>
<thead>
<tr>
<th>Left-wing</th>
<th>Centrist</th>
<th>Right-wing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I &gt; C &gt; D &gt; FC$</td>
<td>$C &gt; D &gt; FC &gt; I$</td>
<td>$D &gt; FC &gt; I &gt; C$</td>
</tr>
</tbody>
</table>

Note: $I = an increase in social service provision; D = a decrease in social service provision; C = a maintenance of current levels of social service provision; FC = future cuts in social service provision; > = "is strictly preferred to."
You will immediately notice that something very strange has happened. Despite the fact that the new alternative receives a lower score than all of the original options and that it is not the first choice of any of the councillors, its addition as an active alternative for consideration changes how the councillors, as a collectivity, rank the three original options. In doing so, it changes the outcome of the vote. Whereas the group had previously been "indifferent" between the three original options, it now possesses a strict and transitive preference ordering over them, with "decreased spending" as the group’s "most preferred" outcome. Note that this is the case despite the fact that none of the councillors has changed the way that he rank orders \( I, D, \) and \( C. \) In effect, the choice that the council now makes has been influenced by the introduction of what might be called an "irrelevant alternative." As this example illustrates, the Borda count does not demonstrate the property that political scientists refer to as "independence from irrelevant alternatives." 

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The Borda count now produces a clear winner! The choice has been influenced by the introduction of what might be called an ‘irrelevant alternative.’

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Points awarded</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left-wing</td>
</tr>
<tr>
<td>Increase spending</td>
<td>3</td>
</tr>
<tr>
<td>Decrease spending</td>
<td>1</td>
</tr>
<tr>
<td>Current spending</td>
<td>2</td>
</tr>
<tr>
<td>Future cuts in spending</td>
<td>0</td>
</tr>
</tbody>
</table>
Decision rules that are not ‘independent of irrelevant alternatives’ allow wily politicians to more easily manipulate the outcome of a decision making process to produce their most preferred outcome.

Rather than making persuasive arguments about the desirability of his most preferred outcome, a politician might get her way by the imaginative introduction of an alternative that has no chance of winning, but that can influence the alternative that is ultimately chosen.
An alternative decision-making mechanism that overcomes the potential instability of majority rule in round-robin tournaments requires actors to begin by considering only a subset of the available pair-wise alternatives.
A voting agenda is a plan that determines the order in which votes occur.

- First round: $I$ vs. $D$.

- Second round: Winner of first round vs. $C$. 
In the contest between C and I, you know that the eventual outcome will be D. If you decide to have a first-round contest between C and I, you know that the eventual outcome will be D. And if you decide to have a first-round contest between C and D, you know that the eventual outcome will be I. Consequently, if one of the councillors is given the power to choose the agenda, she is, effectively, given the power to dictate the outcome of the decision-making process. This phenomenon, in which choosing the agenda is tantamount to choosing which alternative will win, is referred to as the “power of the agenda setter,” and it exists in many institutional settings. In our example, the agenda setter can obtain her most preferred outcome simply by deciding what the order of pair-wise contests should be. For example, the centrist councillor would choose agenda 1 in Table 11.7 if she were the agenda setter; the right-wing councillor would choose agenda 2; and the left-wing councillor would choose agenda 3.

In sum, it is possible to avoid the potential for group intransitivity that arises in majority rule round-robin tournaments by imposing an agenda—by designating which outcomes will be voted on first and which outcome will, in effect, be granted entry into a second round, in which it will compete against the winner of the first round. Unfortunately, the outcome of such a process is extremely sensitive to the agenda chosen, and consequently, either of two things is likely to happen. Either the instability of group decision making shifts from votes on outcomes to votes on the agendas expected to produce those outcomes, or some subset of actors is given power to control the agenda and therefore given considerable influence over the outcome likely to be produced. Thus, one possible explanation for observed policy stability in democracies is that some subset of the decision makers is controlling the agenda in a manner that prevents its preferred outcome from being defeated as part of a cycle of majorities. While this set of events might introduce desired stability to the policymaking process, it does so by sacrificing the notion that democratic outcomes reflect the will of the majority.

### Table 11.7

<table>
<thead>
<tr>
<th>Agenda</th>
<th>1st Round</th>
<th>1st-Round winner</th>
<th>2nd Round</th>
<th>2nd-Round winner</th>
<th>Councillor obtaining her most preferred outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I vs. D</td>
<td>D</td>
<td>D vs. C</td>
<td>C</td>
<td>Centrist councillor</td>
</tr>
<tr>
<td>2</td>
<td>C vs. I</td>
<td>I</td>
<td>I vs. D</td>
<td>D</td>
<td>Right-wing councillor</td>
</tr>
<tr>
<td>3</td>
<td>C vs. D</td>
<td>C</td>
<td>C vs. I</td>
<td>I</td>
<td>Left-wing councillor</td>
</tr>
</tbody>
</table>

Note: I = an increase in social service provision; D = a decrease in social service provision; C = a maintenance of current levels of social service provision.

The agenda setter can get her most preferred outcome. The agenda setter is a dictator!
But should we expect all the councillors to vote sincerely?

A strategic or sophisticated vote is a vote in which an individual votes in favor of a less preferred option because she believes doing so will ultimately produce a more preferred outcome.

A sincere vote is a vote for an individual’s most preferred option.
Agenda 1: $I$ vs. $D$, with winner against $C$.

The councillors know that the second round will involve either $D$ vs. $C$ ($C$ wins) or $I$ vs. $C$ ($I$ wins).

Thus, the councillors know that if $D$ wins the first round, then the outcome will be $C$, and that if $I$ wins the first round, then the outcome will be $I$.

This means that the first round of voting is really a contest between $C$ and $I$ (even if they are voting on $I$ and $D$).
Put yourself in the shoes of the right-wing councillor, $D > I > C$.

If she votes for her preferred option ($D$) in the first round, she will end up with $C$ (her worst preferred option) as the final outcome.

Thus, she has a strong incentive to vote strategically for $I$ in the first round, since this will lead to $I$ (her second preferred option) as the final outcome.

Some analysts find strategic voting lamentable and prefer decision rules that induce sincere voting.
It is possible to avoid the potential for group intransitivity by imposing an agenda.
Unfortunately, the outcome of such a process is extremely sensitive to the agenda chosen, and, consequently, either of two things is likely to happen:

1. The instability of group decision making shifts from votes on outcomes to votes on the agendas expected to produce those outcomes.

2. Some subset of actors is given power to control the agenda and, therefore, considerable influence over the outcome likely to be produced.
Another way in which stable outcomes might be produced is by placing restrictions on the preferences actors might have.

It is possible to convey an individual’s preference ordering in terms of a utility function.

- A utility function is essentially a numerical scaling in which higher numbers stand for higher positions in an individual’s preference ordering.
A single-peaked preference ordering is characterized by a utility function that reaches a maximum at some point and slopes away from this maximum on either side, such that a movement away from the maximum never raises the actor’s utility.
The centrist councillor has single-peaked preferences.
The right-wing councillor did not have single-peaked preferences.
The **median voter theorem** states that the ideal point of the median voter will win against any alternative in a pair-wise majority-rule election if (i) the number of voters is odd, (ii) voter preferences are single-peaked, (iii) voter preferences are arrayed along a single-issue dimension, (iv) and voters vote sincerely.
When voters are arrayed along a single-policy dimension in terms of their ideal points, the **median voter** is the individual who has at least half of all the voters at his position or to his right and at least half of all the voters at his position or to his left.
would be opposed by the centrist and right-wing councillors because any such proposal would be farther from their ideal points than the existing status quo. The right-wing councilor would like to move social spending to the left, toward her own ideal point $D$. Any proposal to do this would now be opposed by the centrist and the left-wing councillors because any such proposal would be farther from their ideal points than the existing status quo. As a result, if the status quo is at the centrist councillor’s ideal point, then it is an equilibrium.

Second, suppose that the status quo level of social service spending is anywhere other than $C$—let’s say somewhere to the left of $C$. This type of scenario is shown in Figure 11.5, with the status quo policy arbitrarily placed at $SQ$ (status quo). In this type of situation, both the centrist and left-wing councillors are likely to propose moving social service spending closer to $C$. Let’s suppose they propose $A$. Proposal $A$ beats the $SQ$ because the left-wing and centrist councillors vote for it and only the right-wing councillor votes against. But is policy $A$ an equilibrium? The answer is no. The left-wing and centrist councillors would like to move social service provision farther to the right, closer to their ideal points. Let’s suppose that they now propose $B$. Proposal $B$ will be adopted because it is closer to the ideal points of both the left-wing and centrist councillors than proposal $A$; the right-wing councillor will vote against the new proposal but will lose. Is proposal $B$ an equilibrium? Again, the answer is no. The right-wing and centrist councillors will now want to move social service provision to the left, closer to their ideal points. Any proposal that is closer to $C$ than $B$ will win with the support of the right-wing and centrist councillors. This process will continue until policy fully converges to the ideal point of the centrist councillor at $C$. Only then will the policy converge.

**Figure 11.4** When All Three Councillors Have Single-Peaked Preference Orderings

Note: $I$ = the ideal point of the left-wing councillor; $C$ = the ideal point of the centrist councillor; $D$ = the ideal point of the right-wing councillor.

$C$ wins.
Any proposals will converge on the position of the median voter, $C$. 

Figure 11.5  Illustrating the Power of the Median Voter
The MVT shows that the difficulties we encountered with Condorcet’s Paradox can be avoided if we are willing to both rule certain preference orderings ‘out of bounds’ and reduce the policy space to a single dimension.
Unfortunately, neither of these restrictions is uncontroversial.

• There is nothing intrinsically troubling about individual preferences that are not single-peaked.

• Many political questions are inherently multi-dimensional.
Unfortunately, neither of these restrictions is uncontroversial.

- There is nothing intrinsically troubling about individual preferences that are not single-peaked.

- Many political questions are inherently multi-dimensional.

What if we increase the number of dimensions?
Labor, capital, and agriculture are deciding how to divide a pot of subsidies from the government’s budget.

Each constituency only cares about maximizing subsidies to its own constituency.

The decision-making situation can be represented by a two-dimensional policy space.
The restriction of politics to a single-issue dimension can also be controversial. This is because many political questions are inherently multidimensional. As an example, consider a situation in which the representatives of three constituencies—labor, capital, and agriculture—are deciding how to divide a pot of subsidies from the government's budget. This decision-making situation can be represented by a two-dimensional policy space in which the percentage of subsidies going to labor is one dimension and the percentage of subsidies going to capital owners is the other; anything left over goes to agriculture. This decision-making situation is depicted in Figure 11.6. The downward-sloping dashed line sets an upper bound on all the possible distributions of subsidies. This limit is necessary because there is a finite amount of resources that can be spent on subsidies. In what follows, we assume that the entire pot of subsidies will be distributed between the three constituencies. At point $L$, all of the subsidies go to labor. At point $C$, all of the subsidies go to capital. And at point $A$, all of the subsidies go to agriculture. Any point along the sloping dashed line between $L$ and $C$ is some distribution of the subsidies between labor and capital; agriculture gets nothing. Any point along the solid vertical line between $L$ and $A$ is some distribution of subsidies between labor and agriculture.
An **indifference curve** is a set of points such that an individual is indifferent between any two points in the set.

The **winset** of some alternative $z$ is the set of alternatives that will defeat $z$ in a pair-wise contest if everyone votes sincerely according to whatever voting rules are being used.
the subsidies between labor and agriculture; capital gets nothing. And any point along the solid horizontal line between \( A \) and \( C \) is some distribution of the subsidies between agriculture and capital; labor gets nothing. Finally, any point within the triangle \( LAC \) is some distribution of the subsidies between all three constituencies. For example, at point \( E \), the subsidies are divided equally between labor, capital, and agriculture.

Imagine that each constituency wants to maximize its share of the government subsidies but has no opinion about how the portion it does not receive is divided among the other constituencies. If each constituency votes to allocate the subsidies by majority rule and can propose a change in the division at any time, then the problem of cyclical majorities that we encountered with Condorcet's paradox will rear its ugly head again. To see why, imagine that someone, perhaps the national government, proposes to divide the subsidies equally between all three constituencies. This point can be thought of as the status quo proposal, and it is marked as \( SQ \) in Figure 11.7. Given the assumptions that we have made, the most preferred outcome for each constituency will be to get 100 percent of the subsidies for itself. Recall that these ideal points are given by points \( L \) (labor), \( A \) (agriculture), and \( C \) (capital) in Figure 11.6.

Two-Dimensional Voting with Winsets

**Figure 11.7** Two-Dimensional Voting with Winsets

<table>
<thead>
<tr>
<th>Percentage of Subsidies to Labor</th>
<th>Percentage of Subsidies to Capitalists</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.0</td>
<td>0</td>
</tr>
<tr>
<td>66.6</td>
<td>33.3</td>
</tr>
<tr>
<td>33.3</td>
<td>66.6</td>
</tr>
<tr>
<td>0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Note: The three solid gray lines going through \( SQ \) (status quo) are the indifference curves for labor (\( L \)), capital (\( C \)), and agriculture (\( A \)); \( P_1 = \text{proposal 1} \). The shaded triangles are winsets that represent alternative divisions of the subsidies that are preferred by a majority to the status quo; the majority in question is shown in each winset.
The proposal is denoted by $P_1$ in Figure 11.7. Because this proposal leaves both agriculture and labor better off vis-à-vis the status quo, the agriculture and labor representatives will vote to accept this proposal; the capital representative will vote against the proposal because capital would be worse off. Hence, proposal $P_1$ will defeat the original status quo 2–1 and become the new status quo proposal. Are there any alternative divisions of the subsidies that a majority of representatives prefer to the new status quo proposal $P_1$? To answer this question, we must draw the indifference curves of the three constituencies with respect to $P_1$ and see if there are any nonempty winsets. We do this in Figure 11.8.

As before, the indifference curves for each constituency are shown by the gray lines going through the new status quo proposal $P_1$. As Figure 11.8 illustrates, there are two winsets. The winset labeled $L + C$ contains alternatives that are preferred to $P_1$ by both labor and capital. The winset labeled $A + C$ contains alternatives that are preferred to $P_1$ by both agriculture and capital.

In other words, there are several alternative divisions of the subsidies that are preferred by a majority to the new status quo proposal $P_1$. For example, the capital representative might propose to give two thirds of the subsidies to labor and one third of the subsidies to capital. This proposal is denoted by $P_2$ in Figure 11.8. Because this proposal leaves labor better off (labor receives 66.6 percent instead of 50 percent) and...
capital better off (capital gets 33.3 percent instead of 0 percent), the labor and capital representatives will vote to accept proposal $P_2$; the agriculture representative will vote against the proposal because agriculture will be worse off (agriculture receives 0 percent instead of 50 percent). Hence, proposal $P_2$ will defeat proposal $P_1$ and become the new status quo proposal.

Is $P_2$ a stable division of subsidies? The answer is no. Agriculture, which is not getting any share of the subsidies under proposal $P_2$, could propose a 50–50 division of the subsidies between itself and capital. This is proposal $P_3$ in Figure 11.9. This proposal would defeat $P_2$ because agriculture would vote for it (agriculture receives 50 percent instead of 0 percent), and capital would also vote for it (capital receives 50 percent instead of 33 percent). Thus, proposal $P_3$ would dislodge proposal $P_2$ as the new status quo proposal. Because there is always some division of the subsidies that gives the excluded constituency a share of the pot while giving one of the other constituencies a bigger share of the pot than it is receiving with the status quo proposal, this process of ever-shifting divisions of the subsidy pot can be expected to go on forever. This is illustrated in Figure 11.9.

The process of cyclical majorities highlighted in Figure 11.9 exemplifies a famously unsettling theorem about politics relating to majority rule in multidimensional settings.

**Figure 11.9**

Two-Dimensional Voting with Cyclical Majorities

<table>
<thead>
<tr>
<th>Percentage of Subsidies to Labor</th>
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<tbody>
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<td>0</td>
</tr>
<tr>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>33.3</td>
<td>66.6</td>
</tr>
</tbody>
</table>

Note: SQ = original status quo; $P_1 = proposal that beats SQ; $P_2 = proposal that beats $P_1$; $P_3 = proposal that beats $P_2$; $P_4 = proposal that beats $P_3$, and so on.
The Chaos Theorem states that if there are two or more issue dimensions and three or more voters with preferences in the issue space who all vote sincerely, then except in the case of a rare distribution of ideal points, there will be no Condorcet winner.
Unless we are lucky enough to have a set of actors who hold preferences that do not lead to cyclical majorities, then either of two things will happen:

1. The decision-making process will be indeterminate and policy outcomes hopelessly unstable.

2. There will exist an actor – the agenda setter – with the power to determine the order of votes in such a way that she can produce her most favored outcome.
Summary So Far

Condorcet’s Paradox shows that a set of rational individuals can form a group that is incapable of choosing rationally in round-robin tournaments.
Alternative voting schemes like the Borda count allow clear winners in some cases, but the outcomes are not necessarily robust.
If we employ ‘single elimination’ tournaments that form a voting agenda, the cyclical majorities may be avoided but whoever controls the agenda can dictate the outcome.
The problem of instability can be overcome if we have a single-issue dimension\textit{ and} each voter has single-peaked preferences.
But why should we restrict people’s preferences and what about multi-dimensional problems?
So, should we just drop majority rule?
Arrow’s Theorem states that every decision-making process that we could possible design must sacrifice at least one of Arrow’s fairness conditions – non-dictatorship, universal admissibility, unanimity, or independence from irrelevant alternatives – if it is to guarantee group transitivity and, hence, stable outcomes.
Arrow presented four fairness conditions that he believed all decision-making processes should meet.
1. The non-dictatorship condition states that there must be no individual who fully determines the outcome of the group decision-making process in disregard of the preferences of the other group members.
2. The **universal admissibility condition** states that individuals can adopt any rational preference ordering over the available alternatives.
3. The **unanimity or pareto optimality condition** states that if all individuals in a group prefer $x$ to $y$, then the group preferences must reflect a preference for $x$ to $y$ as well.

- Basically, the unanimity condition states that if everybody prefers $x$ to $y$, the group should not choose $y$ if $x$ is available.
4. The **independence of irrelevant alternatives condition** states that group choice should be unperturbed by changes in the rankings of irrelevant alternatives.

- Suppose that, when confronted with a choice between \(x, y,\) and \(z,\) a group prefers \(x\) to \(y.\)

- The IIA condition states that if one individual alters their ranking of \(z,\) then the group must still prefer \(x\) to \(y.\)
If we take Arrow’s conditions of unanimity and IIA as uncontroversial, then we face an institutional ‘trilemma’ between stable outcomes, universal admissibility, and non-dictatorship.
Arrow’s Theorem basically states that when designing institutions, we can choose one and only one side of the triangle.

- If we want group rationality and stable outcomes, then we must give up either non-dictatorship or universal admissibility.

- If we want to avoid dictatorship, then we must give up group rationality or universal admissibility.

- If we hold individual preferences inviolable, then we must give up non-dictatorship or group rationality.
Arrow’s Theorem shows that it is difficult to interpret the outcome of any group decision-making process as necessarily reflecting the will of the group.
• When a group comes to a clear decision, it may mean that individual preferences lined up in a way that allowed for a clear outcome that represented the desires of a large portion of the group.

• **But** it may also mean that individuals with inconvenient preferences were excluded from the process, or that some actor exercised agenda control.

• In such cases, outcomes may reflect the interest of some powerful subset of the group rather than the preferences of the group as a whole, or even some majority of the group.
Every decision-making mechanism must grapple with the trade-offs posed by Arrow’s Theorem, and every system of government represents a collection of such decision-making mechanisms.

Thus, we can evaluate different systems of government in terms of how their decision-making mechanisms tend to resolve the trade-offs between group rationality and Arrow’s fairness criteria.

There is no perfect set of decision-making institutions.
A piece of legislation cannot cover all conceivable contingencies for which it might be relevant.

This requires that in any specific instance a judge, bureaucrat, or lawyer must determine whether a specific statute is applicable or not.

Judges often ask, “What did Congress intend in passing this law?”
Liberals (in the American sense) have developed principles of statutory interpretation to enable broad meaning to be read into acts of Congress.

Conservatives, on the other hand, insist on requiring judges to stick to the plain meaning of the statutory language.
But who is right?
But who is right?

Short of appealing to our own prejudices and policy preferences, we can provide an analytical perspective based on Arrow’s Theorem.
Arrow’s Theorem cautions against assigning individual properties to groups. Individuals are rational, but a group is not.

If this is true, how can one make reference to the intent of a group?

Legislators may have an intention, but a legislature does not.

Because groups differ from individuals and may be incoherent, legislative intent is an oxymoron!
The Daily Show and Social Choice Theory here