Strategy and Politics: Rational Choice

Matt Golder

Pennsylvania State University

Notes

Rational Choice Theory

Assumption: Rationality

Rational choice theory (RCT) is a theory of decision making under which a *decision maker* chooses the best *action* (or an action that is at least as good as all others) according to her *preferences* among the set of actions available to her.

No qualitative restriction is placed on the decision-maker's preferences.

Rationality lies in the consistency of her decisions when faced with different sets of available actions, not in the nature of her likes and dislikes.

Notes

Actions

RCT is based on a model with two components: (i) a set A consisting of all the actions that, under some circumstances, are available to the decision-maker and (ii) a specification of the decision-maker's preferences.

In any given situation, the decision-maker is faced with a subset of A ($B \subseteq A$), from which she must choose a single element.

The decision-maker knows this subset of available choices and takes it as given.

Example: Set A could be the set of bundles of goods that the decision-maker can possibly consume. Given her income at any time, she is restricted to chose from the subset of A containing the bundles she can afford.

Notes

Preferences and Payoff Functions

Preferences are descriptions of how people evaluate actions.

Suppose we have $a, b \in B$.

1. **Strict preference relation**
   $aP_ib \equiv a \succ_i b$ means that action a is strictly preferred by person i to action b.
2. **Indifference preference relation**
   $aI_ib \equiv a \sim_i b$ means that person i is indifferent between action a and action b.
3. **Weak preference relation**
   $aR_ib \equiv a \succeq_i b$ means that action a is at least as good as action b for person i.

Notes

Preferences and Payoff Functions

RCT assumes that decision makers have at least a weak preference ordering over the actions available to them i.e. decision makers can at least weakly order their available actions.

A weak preference order R must satisfy two properties:

1. **Completeness (Comparability)**: For every a, $b \in B$, it must be that aR_ib or bR_ia or both. Alternatives are said to be comparable in terms of preferences (and the preference relation complete), if, for any two possible alternatives (say, a and b), either aR_ib or bR_ia or both.
2. **Transitivity**: For every a, b, $c \in B$, if aR_ib and bR_ic, then it must be the case that aR_ic.

If i's preferences satisfy completeness and transitivity, then i is said to possess a preference ordering. The *rational* choice is the action at the top of the ordering.

Notes

Preferences and Payoff Functions

The assumptions of completeness and transitivity yield an ordering principle – they permit an individual to take a set of objects and place them in an order that reflects their preferences.

Rationality is associated with this capacity to order and an aptitude to choose from the top of the order.

No other restriction is imposed on preferences.

In particular, we allow a person's preferences to be altruistic in the sense that how much she likes an outcome depends on some other person's welfare.

Notes

Preferences and Payoff Functions

Not all preference relations satisfy completeness and transitivity.

Comparability requires that comparisons make sense.

- Choices must have meaning to the choosers if they are to be guided by principled considerations such as those associated with rationality.

Transitivity requires that the chooser be consistent, which does not happen all the time.

- Saline-solution and shades-of-gray experiments.
- It is often hard to be consistent when comparisons are difficult (close) and when little is at stake.

Inconsistencies are likely to be high when the stakes are low, uncertainty is high, and individual choices are of little consequence.

Notes

Preferences and Payoff Functions

Two ways to write preferences down:

1. One could use R_i, P_i, and I_i for each possible pair of actions.
2. One could 'represent' the preferences using a *payoff function*, also called a preference indicator function, utility function, or objective function.

A **payoff (utility) function** associates a number with each action such that actions with higher numbers are preferred.

More precisely, the payoff function u represents a decision-maker's preferences if, for any actions a in A and b in A,

$$u(a) > u(b) \text{ if and only if the decision-maker prefers } a \text{ to } b.$$

Notes

Payoff Function: Example

Suppose we have a decision-maker, Sona, whose set of available actions is:

$$B = \{Congress, Legislature, Teach\}$$

Sona's preference ordering is $Congress \succ Legislature \sim Teach$.

A payoff function, u, that represents these preferences might be

$$u(Congress) = 1,\ u(Legislature) = 0,\ u(Teach) = 0$$

Notes

Payoff Function: Example

A payoff function, u, that represents these preferences might be

$$u(Congress) = 1, \; u(Legislature) = 0, \; u(Teach) = 0$$

But how about the following payoff function, v,

$$v(Congress) = 50, \; v(Legislature) = 10, \; v(Teach) = 10$$

or the payoff function, w,

$$w(Congress) = 1000, \; w(Legislature) = -42, \; w(Teach) = -42$$

Notes

Payoff Function: Example

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It turns out that all of these payoff functions represent Sona's preferences.

Notes

Ordinal Preferences

Preferences are ordered according to the perspective of the person making the decision – they provide *ordinal* information.

Preferences and the payoff functions that represent them do not provide information about relative differences.

Example: u is a function such that $u(a) = 0$, $u(b) = 1$, and $u(c) = 100$. Can we infer that individual i likes action c a lot more than actions a and b?

Notes

Ordinal Preferences

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Example: u is a function such that $u(a) = 0$, $u(b) = 1$, and $u(c) = 100$. Can we infer that individual i likes action c a lot more than actions a and b? No.

Notes

Ordinal Preferences

Thus, there are many different payoff functions that can represent a decision-maker's preferences.

If u represents a decision-maker's preferences and v is another payoff function for which

$$v(a) > v(b) \text{ if and only if } u(a) > u(b)$$

then v also represents the decision-maker's preferences.

Notes

Payoff Function: Example

Altruistic preferences: Allocation of dollars (x, y) – x to Sona and y to Sean – where x and $y \in R_{+}$. Let's suppose that Sona values her dollars twice as much as what Sean gets.

How would Sona rank the following allocations: (1, 4), (2, 1), and (0,3)?

We could write Sona's payoff function as $u = 2x + y$.

Table: Altruistic Preferences: $u = 2x + y$

Allocation (x, y)	Payoff	Rank
(1,4)	$u(1,4) = 2(1) + 4 = 6$	1
(2,1)	$u(2,1) = 2(2) + 1 = 5$	2
(0,3)	$u(0,3) = 2(0) + 3 = 3$	3

Notes

Payoff Function: Example

But we could also have written Sona's preferences as $v = x + \frac{y}{2}$ or $w = (x + \frac{y}{2})^2$.

Table: Altruistic Preferences

(x, y)	Payoff	Payoff	Payoff	Rank
	$u = 2x + y$	$v = x + \frac{y}{2}$	$w = (x + \frac{y}{2})^2$	
(1,4)	$u(1,4) = 2(1) + 4 = 6$	$v(1,4) = 1 + \frac{4}{2} = 3$	$v(1,4) = (1 + \frac{4}{2})^2 = 9$	1
(2,1)	$u(2,1) = 2(2) + 1 = 5$	$v(2,1) = 2 + \frac{1}{2} = 2.5$	$v(2,1) = (2 + \frac{1}{2})^2 = 6.25$	2
(0,3)	$u(0,3) = 2(0) + 3 = 3$	$v(0,3) = 0 + \frac{3}{2} = 1.5$	$v(0,3) = (0 + \frac{3}{2})^2 = 2.25$	3

There is an infinite set of functions (increasing in u) that can represent Sona's preferences in this case.

Notes

Summary

Rational Choice Theory: In any given situation, the decision-maker chooses the member of the available subset of A that is best according to her complete and transitive preferences.

Allowing for the possibility that there are several equally attractive best options, the theory of rational choice is that the action chosen by a decision-maker is at least as good, according to her preferences, as every other available action.

Notes

Generalizing to Expected Payoffs

So far we have dealt with deterministic actions (outcomes). A **deterministic** action is one that always produces the same outcome.

But sometimes actions are probabilistic. A **probabilistic** action is one that produces outcomes only with a certain probability.

If an individual is dealing with deterministic actions, then we say she is operating under conditions of **certainty**.

If an individual is dealing with probabilistic actions, then we say she is operating under conditions of **risk**.

Notes

Generalizing to Expected Payoffs

When individuals are dealing with probabilistic actions, we start to talk about them having preferences over lotteries.

If you make an offer for an item on ebay, then given the behavior of other potential buyers, your offer may be accepted with probability $\frac{1}{3}$ and rejected with probability $\frac{2}{3}$.

We refer to a probability distribution over outcomes likes this as a **lottery** over outcomes.

Notes

Generalizing to Expected Payoffs

How can one think about preferences over lotteries? Suppose that aP_ibP_ic.

We might have the following two lotteries:

$$L_1 : Pr(b) = 1$$
$$L_2 : Pr(a) = 0.3, Pr(c) = 0.7$$

Which lottery does the decision maker prefer?

Notes

Generalizing to Expected Payoffs

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We might have the following two lotteries:

$$L_1 : Pr(b) = 1$$
$$L_2 : Pr(a) = 0.3, Pr(c) = 0.7$$

Which lottery does the decision maker prefer?

We *cannot* derive preferences over lotteries from preferences over deterministic actions (outcomes).

The decision maker may prefer L_1 to L_2 or L_2 to L_1 – it will depend on how she ranks *each lottery*.

Notes

Generalizing to Expected Payoffs

We talk about **payoff** functions when dealing with preferences over deterministic outcomes, but **expected payoff** (Bernoulli) functions when dealing with preferences over lotteries.

A lottery over k outcomes can be denoted as $(p_1, \ldots, p_k)$, where p_k is the probability that the k_{th} outcome occurs.

Under certain assumptions, it is possible to construct a payoff function u over deterministic outcomes such that the decision-maker's preference relation over lotteries is represented by the function $U(p_1, \ldots, p_k) = \sum_{k=1}^{K} p_k u(a_k)$, where a_k is the k_{th} outcome of the lottery:

$$\sum_{k=1}^{K} p_k u(a_k) > \sum_{k=1}^{K} p'_k u(a_k)$$

if and only if the decision-maker prefers the lottery $(p_1, \ldots, p_k)$ to the lottery $(p'_1, \ldots, p'_k)$.

Notes

Generalizing to Expected Payoffs

In other words, the decision-maker evaluates a lottery by its *expected payoff* according to the function u, which is known as the decision-maker's Bernoulli payoff function.

When we employ expected payoffs in our model, we often say that individuals have *von Neumann-Morgenstern (vNM) preferences*.

Example: Suppose there are 3 possible deterministic outcomes - \$0, \$1, or \$5 – and that the decision-maker prefers the lottery $(\frac{1}{2}, 0, \frac{1}{2})$ to $(0, \frac{3}{4}, \frac{1}{4})$. This preference over lotteries is consistent with preferences represented by the expected value of the payoff function u for which $u(0) = 0$, $u(1) = 1$, and $u(5) = 4$ because

$$\frac{1}{2} \times 0 + 0 \times 1 + \frac{1}{2} \times 4 > 0 \times 0 + \frac{3}{4} \times 1 + \frac{1}{4} \times 4$$

Notes

Generalizing to Expected Payoffs: Cardinal Payoffs

Preferences represented by the expected value of a Bernoulli payoff function have the advantage that they are completely specified by that payoff function.

Once we know $u(a_k)$ for each possible outcome a_k, we know the decision-maker's preferences over all lotteries.

One of the assumptions required to construct an expected payoff (Bernoulli) function is that the payoffs $u(a_k)$ now represent **cardinal payoffs**. In other words, they indicate exactly how much the decision-maker prefers one outcome to another.

Notes

Generalizing to Expected Payoffs

What's the relationship between expected (Bernoulli) payoff functions and payoff functions?

Suppose that a decision-maker's preferences over lotteries are represented by the expected value of the Bernoulli payoff function u.

Then u is a payoff function that represents the decision-maker's preferences over deterministic outcomes (which are special cases of lotteries, in which a single outcome is assigned probability 1).

Notes

Generalizing to Expected Payoffs

But the converse is not true: if the decision-maker's preferences over deterministic outcomes are represented by the payoff function u, (i.e. the decision-maker prefers a to a' if and only if $u(a) > u(a')$), then u is not necessarily a Bernoulli payoff function whose expected value represents the decision-makers preferences over lotteries.

Example: Suppose a decision-maker prefers \$5 to \$1 to \$0 and prefers the lottery $(\frac{1}{2}, 0, \frac{1}{2})$ to the lottery $(0, \frac{3}{4}, \frac{1}{4})$. Then her preferences over deterministic outcomes are consistent with the payoff function u for which $u(0) = 0$, $u(1) = 3$, and $u(5) = 4$. However, the preferences over lotteries are not consistent with the expected value of this function since

$$\frac{1}{2} \times 0 + 0 \times 3 + \frac{1}{2} \times 4 < 0 \times 0 + \frac{3}{4} \times 3 + \frac{1}{4} \times 4$$

Notes

Generalizing to Expected Payoffs

We saw earlier that if a decision-maker's preferences in a deterministic environment are represented by the payoff function u, then they are represented by any payoff function that is an increasing function of u.

However, the analogous property is not satisfied by expected (Bernoulli) payoff functions.

They are only analogous if we restrict ourselves to linear functions of u.

Lemma: Suppose there are at least three possible outcomes. The expected values of the Bernoulli payoff functions u and v represent the same preferences over lotteries if and only if there exist numbers η and θ with $\theta > 0$ such that $u(x) = \eta + \theta v(x)$ for all x.

Notes

Generalizing to Expected Payoffs: Risk

The assumption that a decision-maker's preferences are represented by the expected value of a payoff function does not restrict her attitudes to risk: a person whose preferences are represented by such a function may like or dislike risk.

We often say that there are 3 types of individuals:

1. A **risk neutral** person is someone who is indifferent between utility for certain and expected utility of equal value.
2. A **risk averse** person is someone who prefers utility for certain to an expected utility of equal value.
3. A **risk acceptant** person is someone who prefers an expected utility to a certain utility of equal value.

Notes

Generalizing to Expected Payoffs: Risk

Suppose an individual chooses between \$1 for sure and a lottery where the expected value is also \$1.

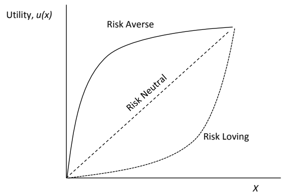
We often say that there are 3 types of individuals:

1. A **risk neutral** person would be indifferent between these two options i.e. the individual would receive the same utility from either choice.
2. A **risk averse** person would receive more utility from the \$1 for sure than the lottery with the expected value of \$1.
3. A **risk acceptant** person would prefer the expected value of \$1 from the lottery to the \$1 for sure i.e. the individual receives more utility from the lottery than the \$1 for sure.

Notes

Generalizing to Expected Payoffs: Risk

Figure: Risk and the Shape of Utility Functions



- Risk neutral – linear utility function
- Risk averse – concave utility function (diminishing marginal utility)
- Risk loving – convex utility function

Notes

Generalizing to Expected Payoffs: Risk

Is Sean risk neutral, risk averse, or risk loving?

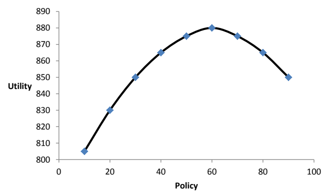
Table: Altruistic Preferences

Policy	Utility
10	805
20	830
30	850
40	865
50	875
60	880
70	875
80	865
90	850

Notes

Generalizing to Expected Payoffs: Risk

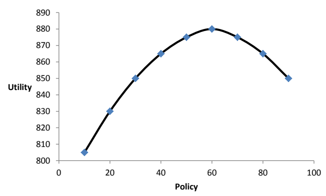
Figure: Sean's Utility Function



Notes

Generalizing to Expected Payoffs: Risk

Figure: Sean's Utility Function



Sean is risk averse.

Notes

Summary

The key idea here is that in probabilistic settings, we need to think about preferences over lotteries and we need to be able to calculate expected payoffs (utility).

Example: We flip a coin. If it is heads I pay you \$1. If it is tails, you pay me \$1. Assume that I prefer more money to less. A (Bernoulli) utility function consistent with these preferences might be $u(x) = x$ i.e. $u(1) = 1$ and $u(-1) = -1$. Am I risk neutral, risk averse, or risk loving?

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Example: We flip a coin. If it is heads I pay you \$1. If it is tails, you pay me \$1. Assume that I prefer more money to less. A (Bernoulli) utility function consistent with these preferences might be $u(x) = x$ i.e. $u(1) = 1$ and $u(-1) = -1$. Am I risk neutral, risk averse, or risk loving?

Risk neutral because I have a linear utility function.

Notes

Summary

The key idea here is that in probabilistic settings, we need to think about preferences over lotteries and we need to be able to calculate expected payoffs (utility).

Example cont'd.: The expected payoff associated with flipping a coin is

$$\frac{1}{2} \times 1 + \frac{1}{2} \times -1 = 0$$

The expected value of an action is just the sum of each payoff from each outcome associated with the action multiplied by the probability that the outcome occurs.

Notes

Summary

In a probabilistic setting, a *rational* decision-maker chooses the action that maximizes the expected payoff (utility).

Example: Suppose I am choosing between flipping a coin with the Bernoulli payoffs from before and obtaining \$0.01 for sure.

Notes

Summary

In a probabilistic setting, a *rational* decision-maker chooses the action that maximizes the expected payoff (utility).

Example: Suppose I am choosing between flipping a coin with the Bernoulli payoffs from before and obtaining \$0.01 for sure.

I would choose to obtain \$0.01 for sure.

Notes

Summary: Note

Remember that the utility numbers are entirely arbitrary and meaningless in and of themselves, the importance is how they relate to each other.

- I could have used numbers where 400 or 200 was the highest in Sean's utility function from before.

We do not do **interpersonal utility comparisons**.

- Suppose that Sean's utility from policy $= 30$ is 500 but Matt's is 1,000. We cannot infer that Matt likes this policy twice as much as Sean.

Notes

School Application Example

High school student wants to go to college to get a good job with a high salary.

Student can apply to:

1. Harvard
2. Penn State
3. East Appalachian State

Student can only afford one application fee.

Notes

School Application Example

Table: School Application Example

Action	Outcome (average salary)
1. Harvard	(a) Graduate: $80,000
	(b) Do not graduate: $0
2. Penn State	(c) Graduate: $40,000
	(d) Do not graduate: $0
3. E. App. State	(e) Graduate: $20,000
	(f) Do not graduate: $0

Student's preference ordering over deterministic outcomes is:

$$aP_icP_ieP_ifI_ibI_ibI_id$$

The student is rational since her preference ordering is:

1. Complete because it accounts for all possible outcomes.
2. Transitive because the student prefers Harvard to East Appalachian State

Notes

School Application Example

We need to specify a utility function to figure out how the student will assign a numerical value (payoff) to each outcome.

Let $u(x) = x$, and assume that we are using vNM preferences.

Table: School Application Example

Action	Outcome (average salary)	Utility [u(x)=x]
1. Harvard	(a) Graduate: $80,000	$80,000
	(b) Do not graduate: $0	0
2. Penn State	(c) Graduate: $40,000	$40,000
	(d) Do not graduate: $0	0
3. E. App. State	(e) Graduate: $20,000	$20,000
	(f) Do not graduate: $0	0

Notes

School Application Example

We now need to know something about the probabilities associated with each outcome.

Note that since the probabilities of mutually exclusive events must sum to 1 i.e. $Pr(a) + Pr(b) = 1$, we can express $Pr(b)$ as $1 - Pr(a)$ or $Pr(a)$ as $1 - Pr(b)$.

Table: School Application Example

Action	Outcome (average salary)	Utility [u(x)=x]	Probability of Outcome
1. Harvard	(a) Graduate: $80,000	$80,000	0.3 (or 30%)
	(b) Do not graduate: $0	0	0.7
2. Penn State	(c) Graduate: $40,000	$40,000	0.75
	(d) Do not graduate: $0	0	0.25
3. E. App. State	(e) Graduate: $20,000	$20,000	0.95
	(f) Do not graduate: $0	0	0.05

Notes

School Application Example

Now we need to calculate the expected utility associated with each action.

$$\begin{aligned} EU_{Harvard} &= [Pr(a) \times u(a)] + [Pr(b) \times u(b)] \\ &= 0.3(80,000) + 0.7(0) \\ &= \$24,000 \end{aligned}$$

$$\begin{aligned} EU_{PSU} &= [Pr(c) \times u(c)] + [Pr(d) \times u(d)] \\ &= 0.75(40,000) + 0.25(0) \\ &= \$30,000 \end{aligned}$$

$$\begin{aligned} EU_{E.App.State} &= [Pr(e) \times u(e)] + [Pr(f) \times u(f)] \\ &= 0.95(20,000) + 0.05(0) \\ &= \$19,000 \end{aligned}$$

Notes

School Application Example

The rational student will apply to the school providing the highest expected utility.

$30,000 is the highest expected utility and so the student should apply to Penn State.

Notes

Columbus Example: Spain's Expected Utility

Spain wanted to have trade with Asia. They had four possible actions. Each action had a certain probability of success.

Table: Columbus Example

Action	Probability of Outcome
1. Go West	P_{west}
2. Go East	P_{east}
3. Go Overland	$P_{overland}$
4. Do Nothing	$P_{nothing}$

Let's assume that $P_{east} = P_{west} = P_{overland} > P_{nothing} = 0$.

Notes

Columbus Example: Spain's Expected Utility

Oftentimes, we express utility in terms of net benefits i.e. $u(a) = b(a) - c(a)$, where $b(a)$ and $c(a)$ are the benefits and costs associated with action a.

Let's assume that the benefits were $b_{east} = b_{west} = b_{overland} > b_{nothing} = 0$.

And let's assume that the costs were $c_{nothing} < c_{west} < c_{east}, \ c_{overland}$.

For the sake of simplicity, let $c_{nothing} = 0$.

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Columbus Example: Spain's Expected Utility

Now we can calculate the expected utility associated with each action.

$$EU_{east} = P_{east}(b_{east} - c_{east}) + (1 - P_{east})(b_{nothing} - c_{east})$$

$$EU_{west} = P_{west}(b_{west} - c_{west}) + (1 - P_{west})(b_{nothing} - c_{west})$$

$$EU_{overland} = P_{overland}(b_{overland} - c_{overland}) + (1 - P_{overland})(b_{nothing} - c_{overland})$$

$$\begin{aligned} EU_{nothing} &= P_{nothing}(b_{nothing} - c_{nothing}) + (1 - P_{nothing})(b_{nothing} - c_{nothing}) \\ &= b_{nothing} \text{ given that } c_{nothing} = 0 \text{ by assumption} \end{aligned}$$

Notes

Columbus Example: Spain's Expected Utility

Given our assumptions about probabilities, benefits, and costs, we know that $EU_{west} > EU_{east}$ and $EU_{west} > EU_{overland}$.

But what is the preference between going west and doing nothing?

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Columbus Example: Spain's Expected Utility

Given our assumptions about probabilities, benefits, and costs, we know that $EU_{west} > EU_{east}$ and $EU_{west} > EU_{overland}$.

But what is the preference between going west and doing nothing?

Spain will choose to go west if:

$$\begin{aligned} EU_{west} &> EU_{nothing} \\ P_{west}(b_{west} - c_{west}) + (1 - P_{west})(b_{nothing} - c_{west}) &> b_{nothing} \\ P_{west}b_{west} - c_{west} - P_{west}b_{nothing} &> 0 \end{aligned}$$

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Solving for c_{west}, we have

$$c_{west} < P_{west}(b_{west} - b_{nothing})$$

Notes

Columbus Example: Spain's Expected Utility

Thus, we have

$$EU_{west} > EU_{nothing}$$

if

$$c_{west} < P_{west}(b_{west} - b_{nothing})$$

Spain will choose to go west if the marginal gain in benefits from going west rather than doing nothing weighted by the probability of success going west is greater than the costs of going west.

Notes

Columbus Example: Portugal's Expected Utility

Let's assume that $P_{east} > P_{west} = P_{overland} > P_{nothing} = 0$.

Let's assume that the benefits were $b_{east} = b_{west} = b_{overland} > b_{nothing} = 0$.

And let's assume that the costs were $c_{nothing}$, $c_{east} < c_{west} < c_{overland}$.

Given these assumptions about probabilities, benefits, and costs, we know that $EU_{east} > EU_{west}$ and $EU_{east} > EU_{overland}$.

$$EU_{east} > EU_{nothing}$$

if

$$c_{east} < P_{east}(b_{east} - b_{nothing})$$

Portugal tried to build trade routes to Asia by going around Africa.

Notes

Columbus Example: Portugal's Expected Utility

Obviously, Portugal would have done better to have funded Columbus' voyage.

But it is important to note that just because someone makes a wrong decision from an *ex post* standpoint does not mean that they are irrational. They may have just been unlucky.

Think of Saddam Hussein's decision to invade Kuwait in 1990.

Notes

Power and Motivation Example: US vs Vietnam

The common definition of **power** is "The ability to get others to do something they would not otherwise do."

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Power and Motivation Example: US vs Vietnam

The common definition of **power** is "The ability to get others to do something they would not otherwise do."

But this definition is not very useful analytically and can produce tautologies.

Example: In the Vietnam war, North Vietnam is able to get the US to do something it would not otherwise do – let North Vietnam take over South Vietnam.

Does this mean that North Vietnam was more powerful than the US?

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Does this mean that North Vietnam was more powerful than the US?

A better definition would define power in some independently observable way.

- The Correlates of War (COW) project has produced an index of power since 1815.
- A better measure might be GNP, but these data are only reliably available since about 1930.

Notes

Power and Motivation Example: US vs Vietnam

The Vietnam War is a classic example of how a weaker power (North Vietnam) can overcome a more powerful opponent (US) as a result of asymmetry of motivation.

Power in international politics is often taken to mean the relative military capabilities or national wealth of two competing countries.

Notes

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Power in international politics is often taken to mean the relative military capabilities or national wealth of two competing countries.

The odds ratios that a particular outcome will occur can be written as:

- The probability that US and S. Vietnam win is $\frac{690+3}{690+3+2} = \frac{693}{695} = 99.7\%$
- The probability that N. Vietnam wins is $\frac{2}{690+3+2} = \frac{2}{695} = 0.03\%$

where the numbers reflect GNP for each country in 1965.

The Vietnam War reminds us that the most powerful countries do not always win – much depends on how motivated the two countries are to bear costs.

Notes

Power and Motivation Example: US vs Vietnam

Suppose there are 3 states, A, B, and C, fighting each other.

B is deciding whether to help A, help C, or stay out of the fight.

Let the national capabilities of the three countries be $A = a$, $B = b$, and $C = c$.

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The odds ratios that a particular outcome will occur can be written as:

- The probability that A wins if B helps A is $P_{BA} = \frac{a+b}{a+b+c}$
- The probability that C wins if B helps C is $P_{BC} = \frac{c+b}{a+b+c}$
- The probability that A wins if B does not help is $P_A = \frac{a}{a+c}$
- The probability that C wins if B does note help is $P_C = \frac{c}{a+c}$

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- The probability that C wins if B does note help is $P_C = \frac{c}{a+c}$

Let's assume that B's utility is U_{BA} for an A victory and U_{BC} for a C victory.

Let's assume that B's costs are K_{BA} for helping A and K_{BC} for helping C.

Notes

Power and Motivation Example: US vs Vietnam

Let's ignore the option of remaining neutral for a moment.

B's expected utility of helping A is

$$EU_{BA} = P_{BA}U_{BA} + (1 - P_{BA})U_{BC} - K_{BA}$$

B's expected utility of helping C is

$$EU_{BC} = P_{BC}U_{BC} + (1 - P_{BC})U_{BA} - K_{BC}$$

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B's expected utility of helping C is

$$EU_{BC} = P_{BC}U_{BC} + (1 - P_{BC})U_{BA} - K_{BC}$$

- If $EU_{BA} > EU_{BC}$, help A.
- If $EU_{BA} < EU_{BC}$, help C.
- If $EU_{BA} = EU_{BC}$, then B is indifferent between helping A and C.

Notes

Power and Motivation Example: US vs Vietnam

B will prefer to help A rather than C if:

$$P_{BA}U_{BA} + (1 - P_{BA})U_{BC} - K_{BA} > P_{BC}U_{BC} + (1 - P_{BC})U_{BA} - K_{BC}$$

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$$P_{BA}U_{BA} + (1 - P_{BA})U_{BC} - K_{BA} > P_{BC}U_{BC} + (1 - P_{BC})U_{BA} - K_{BC}$$

By re-arranging terms, we have

$$\begin{aligned}
(P_{BA} + P_{BC} - 1)(U_{BA} - U_{BC}) &> K_{BA} - K_{BC} \\
\left(\frac{a+b}{a+b+c} + \frac{b+c}{a+b+c} - \frac{a+b+c}{a+b+c}\right)(U_{BA} - U_{BC}) &> K_{BA} - K_{BC} \\
\left(\frac{b}{a+b+c}\right)(U_{BA} - U_{BC}) &> K_{BA} - K_{BC}
\end{aligned}$$

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By re-arranging terms, we have

$$(P_{BA} + P_{BC} - 1)(U_{BA} - U_{BC}) > K_{BA} - K_{BC}$$
$$\left(\frac{a+b}{a+b+c} + \frac{b+c}{a+b+c} - \frac{a+b+c}{a+b+c}\right)(U_{BA} - U_{BC}) > K_{BA} - K_{BC}$$
$$\left(\frac{b}{a+b+c}\right)(U_{BA} - U_{BC}) > K_{BA} - K_{BC}$$

This says that country B will prefer to help country A if:

- $\frac{b}{a+b+c}$ is large i.e. B brings a lot of resources to the fight.
- $U_{BA} - U_{BC}$ is large i.e. B's level of motivation to help A is large.
- $K_{BA} - K_{BC}$ is low i.e. the costs of helping A compared to C are low.

Notes

Power and Motivation Example: US vs Vietnam

Let's assume that B prefers that A wins i.e. $U_{BA} > U_{BC}$.

Let's assume that $K_{BA} - K_{BC} = 1$.

What factors increase the chance that $\left(\frac{b}{a+b+c}\right)(U_{BA} - U_{BC}) > K_{BA} - K_{BC}$?

Notes

Power and Motivation Example: US vs Vietnam

Strength of B

- B is weak (10% of total capabilities)

$$0.1(U_{BA} - U_{BC}) > 1$$

  B will help A only if $U_{BA} - U_{BC} > 10$.

- B is strong (80% of total capabilities)

$$0.8(U_{BA} - U_{BC}) > 1$$

  B will help A only if $U_{BA} - U_{BC} > 1.25$.

As B's power increases relative to the other belligerents, the level of motivation required for B to see one side or the other prevail goes down.

Notes

Power and Motivation Example: US vs Vietnam

If a country is powerful relative to its foes, then it is comparatively easy to participate in wars even when it is not highly motivated.

Thus, the US could fight in the Vietnam war even though it was not particularly motivated by the outcome - no national security interests were at stake.

A country like North Vietnam could only have fought against the US if it was *very* highly motivated to win.

This means that as the costs of war increase above expected levels, those who are least motivated are more likely to give up the fight before those that are highly motivated.

Notes

Power and Motivation Example: US vs Vietnam

Some people confuse defeat with weakness in power.

But victory or defeat depends on power *and the will to win.*

What implications does this have for understanding US policy with respect to Iraq and Afghanistan?

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Power and Motivation Example: US vs Vietnam

Some people confuse defeat with weakness in power.

But victory or defeat depends on power *and the will to win.*

What implications does this have for understanding US policy with respect to Iraq and Afghanistan?

Insurgents, with much to lose, realize that the US has greater potential power but probably believe that the will of the US to endure casualties over the democratization of Iraq is weaker than theirs.

Notes

Power and Motivation Example: Summary

Powerful state are perfectly prepared to fight in wars about which they do not care much, although they are unlikely to continue to fight if they endure high costs in such wars.

Relatively weak states are prepared to fight and endure high costs only in wars about which they care intensely.

Knowing the relative power of rivals is not, by itself, sufficient to anticipate the outcome of a dispute.

Notes

Power and Motivation Example: Addendum

The formulation of the decision problem in the reading is incorrect.

The author – Bruce Bueno de Mesquita – forgot about the option of remaining neutral.

In a choice between helping A or C, we saw that B will prefer to help A if

$$\left(\frac{b}{a+b+c}\right)(U_{BA} - U_{BC}) > K_{BA} - K_{BC}$$

But would B still prefer to help A if the option to remain neutral was available?

Notes

Power and Motivation Example: Addendum

B's expected utility of remaining neutral is:

$$EU_N = P_A U_{BA} + (1 - P_A) U_{BC}$$

B will prefer to help A rather than remain neutral if

$$P_{BA} U_{BA} + (1 - P_{BA}) U_{BC} - K_{BA} > P_A U_{BA} + (1 - P_A) U_{BC}$$

By re-arranging terms, we have

$$(P_{BA} - P_A)(U_{BA} - U_{BC}) > K_{BA}$$

Notes

Power and Motivation Example: Addendum

B will prefer to help A rather than remain neutral if

$$(P_{BA} - P_A)(U_{BA} - U_{BC}) > K_{BA}$$

This says that country B will prefer to help country A if:

- B brings a lot of resources to the fight.
- $U_{BA} - U_{BC}$ is large i.e. B's level of motivation to help A is large.
- The costs of helping A are low.

Notes

Power and Motivation Example: Addendum

Putting it all together, we have that B will prefer to help A if

$$\left(\frac{b}{a+b+c}\right)(U_{BA} - U_{BC}) > K_{BA} - K_{BC}$$

and

$$(P_{BA} - P_A)(U_{BA} - U_{BC}) > K_{BA} \quad (1)$$

- If $U_{BA} = U_{BC}$, B will remain neutral based on Eq. (1).
- If $U_{BA} > U_{BC}$, B will help A or remain neutral depending on how many resources B brings to the fight and the respective costs.
- If $U_{BA} < U_{BC}$, B will help C or remain neutral depending on how many resources B brings to the fight and the respective costs.

Notes

War vs Negotiation Example

The central puzzle about war is that wars occur even though they are costly.

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Scholars have proposed 3 types of explanation for wars:

1. People are sometimes irrational.
2. Leaders who order war enjoy the benefits but do not pay the costs.
3. Rational leaders may prefer to go to war once they weigh up the costs and benefits.

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Scholars have proposed 3 types of explanation for wars:

1. People are sometimes irrational.
2. Leaders who order war enjoy the benefits but do not pay the costs.
3. Rational leaders may prefer to go to war once they weigh up the costs and benefits.

Fearon's paper looks at the the third type of argument – **rationalist or neorealist explanations of war**.

His basic point is that war is *ex post* irrational. Rationalist explanations do not explain why leaders cannot reach an ex ante (pre-war) bargain that would avoid the costs and risks of fighting.

Notes

War vs Negotiation Example

Consider two states, A and B, who have preferences over a set of issues represented by the interval $X = [0, 1]$.

State A prefers issue resolutions closer to 1, while B prefers outcomes closer to 0.

Let the states' utilities for the outcome $x \in X$ be $u_A(x) = x$ and $u_B(1 - x) = 1 - x$.

Without loss of generality, we can set $u_i(1) = 1$ and $u_i(0) = 0$.

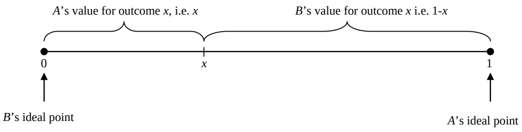
We can think of x as representing the proportion of territory between A and B that is controlled by A.

Although unnecessary, let's assume that the costs of war, c, are the same for both countries.

Notes

War vs Negotiation Example

Figure: Fearon's Model of War



Notes

War vs Negotiation Example

Suppose that if the states fight a war, state A prevails with probability $p \in [0, 1]$, and that the winner gets to choose its ideal point in the issue space.

A's expected utility of war is:

$$EU_A = pu_A(1) + (1 - p)u_A(0) - c = p - c$$

B's expected utility of war is:

$$EU_B = pu_B(0) + (1 - p)u_B(1) - c = 1 - p - c$$

Thus, war can be represented as a costly lottery.

Notes

War vs Negotiation Example

State A will prefer not to go to war whenever $u_A(x) > p - c$.

State B will prefer not to go to war whenever $u_B(1 - x) > 1 - p - c$.

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War vs Negotiation Example

State A will prefer not to go to war whenever $u_A(x) > p - c$.

State B will prefer not to go to war whenever $u_B(1 - x) > 1 - p - c$.

Given these assumptions, it turns out that there is always a set of negotiated settlements that both sides prefer to fighting.

Formally, there exists a subset of X such that for each outcome x in this set, $u_A(x) > p - c$ and $u_B(1 - x) > 1 - p - c$.

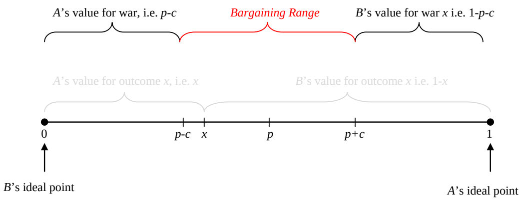
In our case, both states wills strictly prefer any peaceful agreement in the interval $(p - c, p + c)$ to fighting.

The puzzle is why all states don't settle at one of these points rather than go to war.

Notes

War vs Negotiation Example

Figure: Fearon's Model of War



Notes

War vs Negotiation Example

Suppose two people are bargaining over the division of \$100.

For the price of \$20, they can go to war, in which case each player has a 50:50 chance of winning the whole \$100.

The expected value of war is $0.5 \times 100 + 0.5 \times 0 - 20 = \$30$.

This means that neither person should be willing to accept less than \$30 from bargaining.

But there is a range of peaceful, bargained outcomes from (\$31, \$69) to (\$69, \$31) that will make both sides strictly better off than the war option.

Notes

War vs Negotiation Example

The costs and risks of fighting open up a "wedge" of bargained solutions that states will prefer to the gamble of war.

It is the existence of this *ex ante* bargaining range that makes war *ex post* inefficient.

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War vs Negotiation Example

The costs and risks of fighting open up a "wedge" of bargained solutions that states will prefer to the gamble of war.

It is the existence of this *ex ante* bargaining range that makes war *ex post* inefficient.

Three assumptions are required for this result:

1. The states know that there is some true probability p that one state would win in a military contest.
2. The states must be risk neutral or risk averse.
3. There must be a continuous range of peaceful settlements i.e. the issues must be perfectly divisible.

Notes

War vs Negotiation Example

Assumption 1: The states must know that there is some true probability p that one state would win in a military contest.

States may differ in terms of what they think their chances of winning are. This will obscure the boundaries of the bargaining range.

Notes

War vs Negotiation Example

Assumption 1: The states must know that there is some true probability p that one state would win in a military contest.

States may differ in terms of what they think their chances of winning are. This will obscure the boundaries of the bargaining range.

But even if states have private and conflicting estimates of what would happen in a war, if they are rational, then they should know that there can be only one true probability that one or the other will win.

Thus, rational states should still know that there exists a set of agreements that all prefer to a fight.

The question then becomes why states might not reveal information that would avoid war.

Notes

War vs Negotiation Example

Assumption 2: The states must be risk neutral or risk averse.

This assumption basically states that leaders do not like gambling when the downside risk is losing a war.

A risk-acceptant leader must be willing to accept a sequence of gambles that has the expected outcome of eliminating the state and regime.

It seems doubtful that there have been many leaders holding such preferences.

Notes

War vs Negotiation Example

Assumption 3: There must be a continuous range of peaceful settlements.

This assumption requires that the issues in dispute are perfectly divisible so that there are always feasible bargains in the interval $(p - c, p + c)$.

Issues that may not be perfectly divisible might include disputes over land or things like abortion.

We might get around this problem with issue linkage or side payments.

Notes

War vs Negotiation Example

Rationalist Explanation of War 1: The combination of private information about relative power or will and the strategic incentive to misrepresent these.

While states have incentives to locate a peaceful bargain cheaper than war, they also have incentives to do well in the bargaining.

Given the fact of private information about capabilities or resolve, these incentives mean that states cannot always use quiet diplomatic conversations to discover mutually preferable settlements.

The only way to surmount this barrier to communication may be to take actions that produce a real risk of inefficient war.

- States might employ war as a costly signal of privately known but unverifiable information about willingness to fight.
- States might employ war to reveal truthful information about their capabilities.

Notes

War vs Negotiation Example

Rationalist Explanation of War 2: War as a consequence of a credible commitment problem.

Even if states agree on the bargaining range, they may be unable to settle on an efficient bargained outcome when for structural reasons they cannot trust each other to uphold the deal.

All of these explanations rest on there being no third-party enforcer to peaceful bargains.

Notes

War vs Negotiation Example

Rationalist Explanation of War 2: War as a consequence of a credible commitment problem.

Preemptive War: A commitment problem might arise if geography or military technology create a large first-strike or offensive advantage such that a state increases its chance of victory if it attacks rather than defends.

Preventive War: A commitment problem might arise if the strength of state A is likely to grow over time. This is because state A cannot credibly commit to not exploit the greater bargaining leverage that it will have in the future. The result is that state B launches a preventive war.

Strategic Territory: A commitment problem can arise if the objects over which states bargain are themselves sources of military power. For example, territory might provide economic resources that increase bargaining leverage for the state that obtains it, thus causing it to renegotiate any bargain.

Notes

Irrationality of Voting Example

Why don't people turnout more?

Notes

Irrationality of Voting Example

Why don't people turnout more?

Why do they vote at all?

Notes

Irrationality of Voting Example

Why don't people turnout more?

Why do they vote at all?

Suppose we have an election between two candidates, A and E. Sean must decide whether to vote or not.

Suppose that if Sean votes, then the probability that A wins the election is equal to P_{va} and the probability that E wins is $1 - P_{va}$.

And suppose that if Sean does not vote, then the probability that A wins the election is equal to P_{na} and the probability that E wins is $1 - P_{na}$.

Suppose that Sean obtains U_A from an A victory and U_E from a E victory.

Notes

Irrationality of Voting Example

Sean's expected utility from voting is

$$EU_v = p_{va}U_A + (1 - p_{va})U_E$$

Sean's expected utility from not voting is

$$EU_{nv} = p_{na}U_A + (1 - p_{na})U_E$$

Sean's expected benefit from voting is the difference between EU_v and EU_{nv}.

Notes

Irrationality of Voting Example

Sean's benefit from voting is

$$\begin{aligned} EU_v - EU_{nv} &= p_{va}U_A + (1 - p_{va})U_E - p_{na}U_A - (1 - p_{na})U_E \\ & p_{va}U_A + U_E - p_{va}U_E - p_{na}U_A - U_E + p_{na}U_E \\ &= (p_{va} - p_{na})U_A - (p_{va} - p_{na})U_E \\ &= (p_{va} - p_{na})(U_A - U_E) \\ &= \triangle p \times B \end{aligned}$$

$\triangle p = p_{va} - p_{na}$ is the effect of Sean's vote on the probability that A is elected. $B = U_A - U_E$ is the difference in Sean's utility from having A win as opposed to E win.

Sean will vote if the benefits from voting $\triangle p \times B$ are greater than the costs, c.

$$\triangle p \times B > c$$

Notes

Irrationality of Voting Example

What is the probability of Sean's vote being decisive in the election?

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Irrationality of Voting Example

What is the probability of Sean's vote being decisive in the election?

Sean's vote can change the probability that A wins in only two cases: (i) when the election is a tie without Sean's vote and his vote decides the outcome for A, and (ii) when the election is a one vote win for E without Sean's vote and his vote makes the election a tie.

So unless the election is a tie or E is winning by one vote without Sean's vote, then $\triangle p \times B = 0$, regardless of the size of B.

Given that the likelihood of a voter's vote affecting the outcome of the election is close to zero in most situations, a rational voter will not vote.

It is for this reason that we talk about the **irrationality of voting**.

Notes

Irrationality of Voting Example

But people do vote!

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Thus far we have assumed that voters are instrumentally motivated – they vote to affect the outcome of the election.

If voters are *individually* instrumentally motivated, then they will not vote.

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Irrationality of Voting Example

But people do vote!

Thus far we have assumed that voters are instrumentally motivated – they vote to affect the outcome of the election.

If voters are *individually* instrumentally motivated, then they will not vote.

But this ignores the **consumption** or **expressive benefits** of voting.

Voters may derive some benefits from the act of voting independent of the outcome of the election.

Individuals will vote if

$$\triangle p \times B + D > c$$

Notes

Irrationality of Voting Example

Although appealing as an explanation, consumption or expressive benefits are not a very satisfactory solution to the irrationality of voting.

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Irrationality of Voting Example

Although appealing as an explanation, consumption or expressive benefits are not a very satisfactory solution to the irrationality of voting.

It may be a "theory" of why people vote, but it isn't very useful unless we have an understanding of why some voters obtain these consumption and expressive benefits but others do not, or why the value of these benefits vary across time.

Why do some people have a "taste" for voting but others do not?

Notes