Strategy and Politics: Strategic (Normal) Form Games

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Notes

Interacting Decision-Makers

So far, the decision-maker chooses an action from a set A and cares only about this action. We refer to the study of such situations as **decision theory**.

A decision-maker in the world often does not have the luxury of controlling all the variables that affect her. If some of the variables that affect her are the actions of other decision-makers, then her decision-making problem is altogether more challenging than that of an isolated decision-maker.

The study of such situations is referred to as (non-cooperative) **game theory**.

Notes

Strategic Form Games

A strategic or normal form game is a particular type of model of interacting decision-makers.

In recognition of the interaction, we refer to the decision-makers as players.

Each player has a set of possible actions.

Interaction between the players is captured by allowing each player to be affected by the actions of all players, not only her own action.

Specifically, each player has preferences about **action profiles** – the list of all combinations of players' actions.

Notes

Action Profiles

Suppose we have two players, Jeff and Thomas.

The set of actions for each player, A_i, are:

- $A_{Jeff} = \{$stand up, sit down$\}$
- $A_{Thomas} = \{$run, walk$\}$

An action profile, a, is a combination of players' actions, i.e. (stand up, run).

The set of action profiles, **a**, is the list of all combinations of players' actions.

- $\{$(stand up, run), (stand up, walk), (sit down, run), (sit down, walk)$\}$

In a strategic game, players have preferences over action profiles.

Notes

Strategic Form Game

A **strategic form game** (with ordinal payoffs) consists of

1. a set of players, N
2. for each player i, a set of actions, A_i
3. for each player i, preferences over the set of action profiles.

Time is absent in the model: each player chooses her action once and for all, and the players choose their actions "simultaneously" in the sense that no player is informed, when she chooses her action, of the action chosen by any other player.

Notes

Prisoner's Dilemma

Two suspects in a major crime are held in separate cells.

There is enough evidence to convict each of them of a minor offense, but not enough evidence to convict either of them of the major crime unless one of them acts as an informer against the other ("finks").

If they both stay quiet, each will be convicted of the minor offense.

If one and only one of them finks, she will be freed and used as a witness against the other, who will be convicted of the major crime.

If they both fink, each will be convicted of the major crime but some consideration will be taken into account for their cooperation.

This is your standard "Law and Order" scenario.

Notes

Prisoner's Dilemma

The setup of the Prisoner's Dilemma is

- Players: $N = \{1, 2\}$
- Actions: $A_1 =$

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- Players: $N = \{1, 2\}$
- Actions: $A_1 = \{$fink, quiet$\}$, $A_2 = \{$fink, quiet$\}$, or $A_i = \{$fink, quiet$\}$ for $i = 1, 2$.
  - The set of action profiles are $a = \{$(FF), (FQ), (QF), (QQ)$\}$, where the first action belongs to player 1 and the second action belongs to player 2.
- Preferences

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  - The set of action profiles are $a = \{$(FF), (FQ), (QF), (QQ)$\}$, where the first action belongs to player 1 and the second action belongs to player 2.
- Preferences
  - Player 1: $FQ > QQ > FF > QF$ or ${}_{FQ}P_{QQ}P_{FF}P_{QF}$
  - Player 2: $QF > QQ > FF > FQ$ or ${}_{QF}P_{QQ}P_{FF}P_{FQ}$

Notes

Prisoner's Dilemma

As with decision theory, it is frequently convenient to specify the players' preferences by giving payoff functions that represent them.

There are many payoff functions that we could use to capture the preferences in the Prisoner's Dilemma.

$$u_1 = \begin{cases} 3 & \text{if } FQ \\ 2 & \text{if } QQ \\ 1 & \text{if } FF \\ 0 & \text{if } QF \end{cases}$$

$$u_2 = \begin{cases} 3 & \text{if } QF \\ 2 & \text{if } QQ \\ 1 & \text{if } FF \\ 0 & \text{if } FQ \end{cases}$$

Notes

Prisoner's Dilemma

A convenient way of showing a strategic form game with 2 or 3 players is in the form of a matrix or table.

Figure: Prisoner's Dilemma

		Player 2	
		Q	F
Player 1	Q	2, 2	0,3
	F	3,0	1,1

The Prisoner's Dilemma game is useful for modeling situations where there are certain gains from cooperation but also certain disadvantages.

Notes

Pure Coordination

The setup of a pure coordination game is

- Players: $N = \{1, 2\}$
- Actions: $A_1 = \{\text{left, right}\}$, $A_2 = \{\text{left, right}\}$, or $A_i = \{\text{left, right}\}$ for $i = 1, 2$.

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  - The set of action profiles are $a = \{$(LL), (LR), (RL), (RR)$\}$, where the first action belongs to player 1 and the second action belongs to player 2.
- Preferences

  - Player 1: $LL = RR > LR = RL$
  - Player 2: $LL = RR > LR > RL$

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Pure Coordination

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$$u_1 = \begin{cases} 1 & \text{if } LL \\ 0 & \text{if } LR \\ 0 & \text{if } RL \\ 1 & \text{if } RR \end{cases}$$

$$u_2 = \begin{cases} 1 & \text{if } LL \\ 0 & \text{if } LR \\ 0 & \text{if } RL \\ 1 & \text{if } RR \end{cases}$$

Notes

Pure Coordination

Figure: Pure Coordination

		Player 2: L	Player 2: R
Player 1	L	1, 1	0,0
	R	0,0	1,1

A pure coordination game is useful for modeling situations where there are symmetric gains from cooperation.

Notes

Battle of the Sexes: Asymmetric Coordination

The setup of a Battle of the Sexes game is

- Players: $N = \{1, 2\}$
- Actions: $A_1 = \{$boxing, ballet$\}$, $A_2 = \{$boxing, ballet$\}$, or $A_i = \{$boxing, ballet$\}$ for $i = 1, 2$.

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- Players: $N = \{1, 2\}$
- Actions: $A_1 = \{$boxing, ballet$\}$, $A_2 = \{$boxing, ballet$\}$, or $A_i = \{$boxing, ballet$\}$ for $i = 1, 2$.
    - The set of action profiles are $a = \{$(boxing, boxing), (boxing, ballet), (ballet, boxing), (ballet, ballet)$\}$.
- Preferences

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- Actions: $A_1 = \{$boxing, ballet$\}$, $A_2 = \{$boxing, ballet$\}$, or $A_i = \{$boxing, ballet$\}$ for $i = 1, 2$.
    - The set of action profiles are $a = \{$(boxing, boxing), (boxing, ballet), (ballet, boxing), (ballet, ballet)$\}$.
- Preferences
    - Player 1: (boxing; boxing)$>$ (ballet; ballet)$>$(boxing; ballet)$>$(ballet; boxing)
    - Player 2: (ballet; ballet)$>$(boxing; boxing)$>$(boxing; ballet)$>$(ballet; boxing)

Notes

Battle of the Sexes: Asymmetric Coordination

There are many payoff functions that we could use to capture the preferences in an asymmetric coordination game.

$$u_1 = \begin{cases} 3 & \text{if (Boxing; Boxing)} \\ 2 & \text{if (Ballet; Ballet)} \\ 1 & \text{if (Boxing; Ballet)} \\ 0 & \text{if (Ballet; Boxing} \end{cases}$$

$$u_2 = \begin{cases} 3 & \text{if (Ballet; Ballet)} \\ 2 & \text{if (Boxing; Boxing)} \\ 1 & \text{if (Boxing; Ballet)} \\ 0 & \text{if (Ballet; Boxing} \end{cases}$$

Notes

Battle of the Sexes: Asymmetric Coordination

Figure: Asymmetric Coordination

		Player 2	
		Boxing	Ballet
Player 1	Boxing	3, 2	1, 1
	Ballet	0, 0	2,3

An asymmetric coordination game is useful for modeling situations where there are gains from cooperation but also mild competition.

Notes

Matching Pennies

Figure: Matching Pennies

		Player 2: Heads	Player 2: Tails
Player 1	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

A matching pennies game is a **zero-sum game** and can be used to model situations of strict competition.

Notes

Nash Equilibrium Solution Concept

We now need a theory about how games are played – Nash equilibrium.

A **Nash equilibrium** in a game with ordinal preferences is an action profile (a^*) such that for each player i

$$u_i(a^*) \geq u_i(a_i, a^*_{-i})$$

We can write down Nash equilibria in terms of either action profiles or best response functions.

Notes

Nash Equilibrium Solution Concept

There are two components to the NE solution concept.

1. **Rationality** – individuals choose the best available action
2. **Beliefs** – individuals require beliefs about how others will play
   - In a game, the best available action for any given player depends, in general, on the other players' actions.
   - Hence, when choosing an action a player must have in mind the actions the other players will choose. That is, she must form a belief about the other players' actions.

But where do the beliefs come from?

Notes

Nash Equilibrium Solution Concept: Beliefs

Beliefs

- The general idea is that each player's belief is derived from her past experience playing the game, and that this experience is sufficiently extensive that she knows how her opponents will play. Since this is true for each player, they share coordinated beliefs about the game.
- Although we assume that each player has experience playing the game, we assume that she views each play of the game in isolation. She does not condition her action on the particular opponent she is playing or expect her current action to affect the future behavior of others.
- This becomes more realistic if we imagine that there is a population of Player 1s and a population of Player 2s etc. In each play of the game players are selected randomly, one from each population.
- Thus, each player engages in the game repeatedly, but with ever-changing opponents. Her experience leads her to beliefs about the actions of "typical" opponents, not any specific set of opponents.

Notes

Nash Equilibrium Solution Concept: Steady State

Steady State Interpretation

- In the idealized setting in which the players in any given play of the game are drawn randomly from a collection of populations, then a NE corresponds to a steady state.
- If, whenever the game is played, the action profile is the same as the NE a^*, then no player has a reason to choose any action different from her component of a^* i.e. no incentive to deviate.
- A NE embodies a stable "social norm": if everyone else adheres to it, no individual wishes to deviate from it.

Note that none of this tells us how you get to the equilibrium, just that if you are in an equilibrium, you will stay there.

The key is that there is no gain from unilaterally deviating.

Notes

Prisoner's Dilemma

Figure: Prisoner's Dilemma

		Player 2	
		Q	F
Player 1	Q	2, 2	0,3
	F	3,0	1,1

- $a^* =$ (QQ)?
- $a^* =$ (FQ)?
- $a^* =$ (QF)?
- $a^* =$ (FF)?

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Prisoner's Dilemma

Figure: Prisoner's Dilemma

		Player 2 Q	Player 2 F
Player 1	Q	2, 2	0,3
	F	3,0	1,1

- $a^* =$ (QQ)? No
- $a^* =$ (FQ)?
- $a^* =$ (QF)?
- $a^* =$ (FF)?

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- $a^* =$ (QQ)? No
- $a^* =$ (FQ)? No
- $a^* =$ (QF)? No
- $a^* =$ (FF)? Yes The NE in a PD game is {Fink; Fink}

Notes

Pareto Efficiency

Figure: Prisoner's Dilemma

		Player 2 Q	Player 2 F
Player 1	Q	2, 2	0,3
Player 1	F	3,0	1,1

What is odd about the NE (FF)?

Notes

Pareto Efficiency

Figure: Prisoner's Dilemma

		Player 2 Q	Player 2 F
Player 1	Q	2, 2	0,3
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What is odd about the NE (FF)?

There is an outcome that *both* players prefer to the NE outcome.

Pareto efficiency - no player can be made better off without making the other player worse off. FF is not pareto efficient.

FF is **pareto dominated** by QQ in that QQ makes at least one player better off and nobody worse off. We say that FF is **pareto inferior**.

Notes

Pareto Efficiency

Figure: Prisoner's Dilemma

		Player 2 Q	F
Player 1	Q	2, 2	0,3
	F	3,0	1,1

What outcomes in the PD are pareto efficient (or pareto optimal)?

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Pareto Efficiency

Figure: Prisoner's Dilemma

		Player 2 Q	F
Player 1	Q	2, 2	0,3
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What outcomes in the PD are pareto efficient (or pareto optimal)?

(QQ), (FQ), and (QF) are all pareto efficient. Only (FF) is pareto inefficient.

Notes

Prisoner's Dilemma

Figure: Prisoner's Dilemma

		Player 2 Q	F
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Why can't QQ be sustained as an equilibrium?

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Figure: Prisoner's Dilemma

		Player 2	
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Why can't QQ be sustained as an equilibrium?

There is an enforcement problem - neither player can credibly commit not to defect if the other player chooses Q.

Many situations in politics like this – trade agreements, environmental agreements, arms control agreements.

Notes

Pure Coordination

Figure: Pure Coordination

		Player 2	
		L	R
Player 1	L	1, 1	0,0
	R	0,0	1,1

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- $a^* =$ (LL)? Yes
- $a^* =$ (LR)? No
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2 NE in a pure coordination game {L; L} and {R; R}

Notes

Asymmetric Coordination

Figure: Asymmetric Coordination

		Player 2 Boxing	Ballet
Player 1	Boxing	3, 2	1, 1
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- $a^* =$ (boxing; boxing)?
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		Player 2: Heads	Player 2: Tails
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- $a^* =$ (HH)?
- $a^* =$ (HT)?
- $a^* =$ (TH)?
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- $a^* =$ (TH)? No
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There are no NE (in pure strategies).

Notes

Strict and Nonstrict Equilibria

Figure: Unique NE, but not a Strict Equilibrium

		Player 2: L	Player 2: M	Player 2: R
Player 1	T	1, 1	1,0	0,1
	B	1,0	0,1	1,0

The NE is unique – (T; L) – but it is not a strict equilibrium.

An action profiles a^* is a strict Nash equilibrium if for every player i we have

$$u_i(a^*) > u_i(a_i, a^*_{-i})$$

Notes

Best Response Functions

When each player has only a few actions, it is possible to examine each action profile to determine whether it is a NE.

However, this gets more time consuming or impossible, when games get more complicated.

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However, this gets more time consuming or impossible, when games get more complicated.

When this happens, it is often best to work with the "best response functions" of each player.

We denote player i's best response to all players j's actions, where $j \neq i$, as

$$B_i(a_{-i}) = \{a_i \in A_i : u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}) \ \forall \ a_i' \in A_i\}$$

This basically says that if everyone else does a_{-i}, you can't do any better than a_i i.e. a_i is the best you can do given what everyone else is doing.

Notes

Best Response Functions

Rather than define NE in terms of action profiles, we can define NE in terms of best response functions.

The action profile a^* is a Nash equilibrium of a strategic game with ordinal preferences if and only if every player's action is a best response to the other players' actions.

$$a_i^* \in B_i(a_{-i}^*) \ \forall i$$

In other words, in a NE, everyone must be playing a best response.

Notes

Best Response Functions: Example

Figure: Asymmetric Coordination

Player 1		Player 2: Boxing	Player 2: Ballet
	Boxing	3, 2	1, 1
	Ballet	0, 0	2,3

In this 2 person game, $a^* = (a_1; a_2)$.

$$B_1 = \begin{cases} \text{Boxing} & \text{if } a_2 = \text{Boxing} \\ \text{Ballet} & \text{if } a_2 = \text{Ballet} \end{cases}$$

$$B_2 = \begin{cases} \text{Boxing} & \text{if } a_1 = \text{Boxing} \\ \text{Ballet} & \text{if } a_1 = \text{Ballet} \end{cases}$$

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To be a NE, each action a_i must be in the best response function for each player.

- $a^* =$ (boxing; boxing)?
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- $a^* =$ (ballet; ballet)? Yes   2 NE: {boxing; boxing} and {ballet; ballet}

Notes

Best Response Functions

If we can use a payoff matrix to represent the strategic form game, using best response functions is easy.

We simply use stars, circles, or underlining to indicate the best response functions for each player.

We then look for pairs of actions where this is satisfied for each player.

Notes

Best Response Functions: PD

Figure: PD

		Player 2: Q	Player 2: F
Player 1	Q	2, 2	0,3
	F	**3**,0	**1**,1

This is the best response function for Player 1.

Notes

Best Response Functions: PD

Figure: PD

		Player 2: Q	Player 2: F
Player 1	Q	2, 2	0,**3**
	F	**3**,0	**1**,**1**

The best response function for Player 1 is in black and the best response function for Player 2 is in red.

To identify any NE, we simply look for action profiles where each player is playing a best response i.e. (Fink; Fink).

Notes

Best Response Functions: Example

Figure: 3× 3 Game

		Player 2: L	Player 2: C	Player 2: R
Player 1	T	1, 2	2, 1	1, 0
	M	2, 1	0, 1	0, 0
	B	0, 1	0, 0	1, 2

Notes

Best Response Functions: Example

Figure: 3× 3 Game

		Player 2 L	Player 2 C	Player 2 R
Player 1	T	1, 2	2, 1	1, 0
Player 1	M	2, 1	0, 1	0, 0
Player 1	B	0, 1	0, 0	1, 2

This is the best response function for Player 1.

Notes

Best Response Functions: Example

Figure: 3× 3 Game

		Player 2 L	Player 2 C	Player 2 R
Player 1	T	1, 2	2, 1	1, 0
Player 1	M	2, 1	0, 1	0, 0
Player 1	B	0, 1	0, 0	1, 2

There are 2 NE: (M;L) and (B; R).

Notes

Best Response Functions: Infinite Set of Actions

Example: Synergistic Relationship
Two individuals are involved in a synergistic relationship – if both individuals devote more effort to the relationship, they are both better off.

- Players: The two individuals.
- Actions: Each player's set of actions is the set of (non-negative) effort levels that each individual exerts.
- Preferences: Player i's preferences are represented by the payoff function $a_i(c + a_j - a_i)$, for $i = 1, 2$, where a_i is the effort level of individual i, a_j is the effort level of the other individual, and c is a constant.

Each player has infinitely many actions and so we cannot present the game in a table.

How do we find the NE?

Notes

Best Response Functions: Infinite Set of Actions

Example: Synergistic Relationship
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Each player has infinitely many actions and so we cannot present the game in a table.

How do we find the NE? We find each player's best response function.

Notes

Best Response Functions: Infinite Set of Actions

The utility for player 1 is

$$\begin{aligned} u_1 &= a_1(c + a_2 - a_1) \\ &= -a_1^2 + a_1 c + a_1 a_2 \end{aligned}$$

Player 1 wants to choose a_1 that maximizes his utility – his best response.

We can do this with a little calculus.

$$\frac{\partial u_1}{\partial a_1} = -2a_1 + c + a_2$$

Set this equal to 0 and then solve for a_1.

$$\begin{aligned} -2a_1 + c + a_2 &= 0 \\ a_1^* &= \frac{1}{2}(a_2 + c) = b_1(a_2) \end{aligned}$$

Notes

Best Response Functions: Infinite Set of Actions

By symmetry, we have

- $b_1(a_2) = \frac{1}{2}(a_2 + c)$
- $b_2(a_1) = \frac{1}{2}(a_1 + c)$

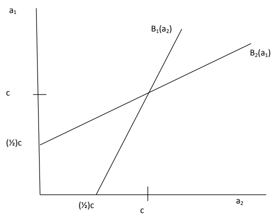
$$\begin{aligned} a_1^* &= \frac{1}{2}(a_2 + c) \\ &= \frac{1}{2}\left[\left(\frac{1}{2}(a_1 + c)\right) + c\right] \\ &= \frac{1}{4}a_1 + \frac{3}{4}c \\ &= c \end{aligned}$$

By symmetry, the NE is $(a_1 = c, a_2 = c)$.

Notes

Best Response Functions: Infinite Set of Actions

Figure: Best Response Functions: Infinite Set of Actions



By symmetry, the NE is $(a_1 = c,\ a_2 = c)$.

Notes

Dominated Actions

In any game, a player's action "strictly dominates" another action if it is superior, no matter what the other players do.

In a strategic game with ordinal preferences, player i's action a''_{ii} strictly dominates her action a'_i if

$$u_i(a''_i, a_{-i}) > u_i(a'_i, a_{-i}) \text{ for every list } a_{-i} \text{ of the other players' actions,}$$

where u_i is a payoff function that represents player i's preferences.

We say that action a'_i is **strictly dominated**.

Notes

Dominated Actions

Figure: PD

		Player 2: Q	Player 2: F
Player 1	Q	2, 2	0,3
	F	3,0	1,1

In the PD, Fink strictly dominates the action Quiet for both players.

We care about this because a strictly dominated action cannot be part of any NE because a strictly dominated action is not a best response to any actions of the other players.

Notes

Dominated Actions

Figure: Dominated Actions

		Player 2	
		L	R
Player 1	T	1	0
	M	2	1
	B	1	3

In this example, action M strictly dominates action T but does not strictly dominate B.

Notes

Dominated Actions

Figure: Dominated Actions

		Player 2	
		L	R
Player 1	T	1	0
	M	2	1
	B	3	2

In this example, action M strictly dominates action T but action B strictly dominates both M and T.

Since action B strictly dominates all other actions, B is a **dominant strategy**.

Notes

Eliminating Strictly Dominated Actions

If we have a strictly dominated action, we can eliminate it because it will not be part of a NE.

Figure: Eliminating Strictly Dominated Actions

		Player 2	
		Q	F
Player 1	Q	2,2	0,3
	F	3,0	1,1

Once we do this in the Prisoner's Dilemma, we see that the only possible NE is $(Fink; Fink)$.

Notes

Eliminating Strictly Dominated Actions

In the following game, R is strictly dominated by L for Player 1. No action is strictly dominated for Player 2.

Figure: Eliminating Strictly Dominated Actions

		Player 2 L	Player 2 R
Player 1	L	1, 1	1,0
	~~R~~	~~0,0~~	~~0,0~~

Once we eliminated R for Player 1, we only need to see what Player 2's best response is to Player 1 playing L. The NE is $(L; L)$.

Notes

Dominated Actions

In any game, a player's action "weakly dominates" another action if the first action is at least as good as the second action, no matter what the other players do, and is better than the second action for some actions of the other players.

In a strategic game with ordinal preferences, player i's action a''_{ii} weakly dominates her action a'_i if

$u_i(a''_i, a_{-i}) \geq u_i(a'_i, a_{-i})$ for every list a_{-i} of the other players' actions,

and

$u_i(a''_i, a_{-i}) > u_i(a'_i, a_{-i})$ for some list a_{-i} of the other players' actions,

where u_i is a payoff function that represents player i's preferences.

We say that action a'_i is **weakly dominated**.

Notes

Dominated Actions

Figure: Dominated Actions

		Player 2 L	Player 2 R
	T	1	0
Player 1	M	2	0
	B	2	1

- Action B strictly dominates action T.
- Action B weakly dominates action M.
- Action M weakly dominates action T.

Notes

Dominated Actions

We've seen that no strictly dominated action can be part of a NE, but what about a weakly dominated strategy?

Notes

Dominated Actions

We've seen that no strictly dominated action can be part of a NE, but what about a weakly dominated strategy?

Figure: Dominated Actions

		Player 2	
		L	R
Player 1	L	1, 1	0,0
	R	0,0	0,0

There are two NE: (L; L) and (R; R).

Action L weakly dominates action R. But R, which is weakly dominated by action L, is part of the NE (R; R).

Notes

Some Additional Examples

Golden Balls, click here

Notes

Some Additional Examples

Golden Balls, click here

Is this a prisoner's dilemma?

What does the game look like?

What are the Nash equilibria?

Notes

Some Additional Examples

Golden Balls: A Modified Prisoner's Dilemma Game

Golden Balls

		B: Split	B: Steal
A	Split	50, 50	0, 100
A	Steal	100, 0	0, 0

- Golden Balls I, click here
- Golden Balls II, click here

Notes

Some Additional Examples

FIGURE 4.12 **A Game of Chicken: The Tractor Face-Off**

		Chuck: Swerve	Chuck: Drive Straight
Ren	Swerve	3, 3	2, 4
Ren	Drive Straight	4, 2	1, 1

Note: Ren's (the row player's) payoffs are shown first in each cell; Chuck's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

- Tractor Faceoff, click here
- Deficit Reduction 2011 (2:57-16:56), click here

Notes

Counterterrorism

Terrorism is the premeditated use or threat of use of violence by individuals or subnational groups to obtain political, religious, or ideological objectives through intimidation of a large audience usually beyond that of the immediate victims.

On September 11, 2001, 19 terrorists affiliated with al-Qaeda hijacked four commercial passenger jets and flew them into various American landmarks (the World Trade Center in New York City and the Pentagon in Washington D.C.) in a series of coordinated terrorist attacks.

Since 9/11, governments around the world have spent tens of billions of dollars on a variety of counterterrorism policies.

Counterterrorism policies generally fall into two types: (i) preemption and (ii) deterrence.

Notes

Counterterrorism

Preemption

- Preemption involves *proactive* policies such as destroying terrorist training camps, retaliating against state sponsors of terrorism, infiltrating terrorist groups, freezing terrorist assets etc.'
- The goal of preemption is to curb future terrorist attacks.
- Preemption makes all countries that are potential targets safer.

Deterrence

- Deterrence involves *defensive* policies such as placing bomb-detectors in airports, fortifying potential targets, and securing borders.
- The goal of deterrence is to deter an attack by either making success more difficult or increasing the likely negative consequences for the terrorists.
- Deterrence often ends up displacing terrorist attacks away from the country taking defensive measures to other countries where targets are not relatively softer.

Notes

Counterterrorism

In a 2005 article entitled, "Counterterrorsim: A Game-Theoretic Analysis", Arce and Sandler use strategic form games to examine these two types of counterterrorism policies.

They argue that governments around the world over-invest in deterrence policies at the expense of preemption policies and that this results in an outcome that is socially suboptimal from the perspective of world security.

Imagine that the United States (US) and the European Union (EU) must decide whether to preempt a terrorist attack or do nothing.

Terrorists are a "passive player" in this game and will attack the weaker of the two targets.

Notes

Preemption Game

Let's suppose that each preemptive action provides a public benefit worth 4 to the US and the EU.

- Recall that preemptive action increases the safety of all countries.

Preemptive action comes at a private cost of 6 to the preemptor

- If only the US (EU) preempts, then the US (EU) will get -2 i.e. $4-6$ and the EU (US) will get 4.
- If the US and EU do nothing, then they each get 0.
- If the US and EU both preempt, then they each receives a payoff of 2 i.e. 8-6.

Notes

Preemption Game

The setup of the preemption game is

- Players: $N = \{US, EU\}$
- Actions: $A_i = \{$preempt, do nothing$\}$ for $i = US,\ EU$.
- Preferences
  - US: (Do Nothing; Preempt)$>$(Preempt; Preempt)$>$(Do Nothing; Do Nothing)$>$(Preempt; Do Nothing)
  - EU: (Preempt; Do Nothing)$>$(Preempt; Preempt)$>$(Do Nothing; Do Nothing)$>$(Do Nothing; Preempt)

Notes

Preemption Game

Figure 4.19 Counterterrorism Preemption Game

		European Union	
		Preempt	Do nothing
United States	Preempt	2, 2	–2, 4
	Do nothing	4, –2	0, 0

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy?

Notes

Preemption Game

FIGURE 4.19 **Counterterrorism Preemption Game**

		European Union: Preempt	European Union: Do nothing
United States	Preempt	2, 2	−2, 4
	Do nothing	4, −2	0, 0

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy? Yes, they both have a dominant strategy to Do Nothing.

What is the NE?

Notes

Preemption Game

FIGURE 4.19 **Counterterrorism Preemption Game**

		European Union: Preempt	European Union: Do nothing
United States	Preempt	2, 2	−2, 4
	Do nothing	4, −2	0, 0

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy? Yes, they both have a dominant strategy to Do Nothing.

What is the NE? (Do Nothing; Do Nothing)

Is (Do Nothing; Do Nothing) pareto efficient?

Notes

Preemption Game

FIGURE 4.19 **Counterterrorism Preemption Game**

		European Union: Preempt	European Union: Do nothing
United States	Preempt	2, 2	−2, 4
	Do nothing	4, −2	0, 0

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy? Yes, they both have a dominant strategy to Do Nothing.

What is the NE? (Do Nothing; Do Nothing)

Is (Do Nothing; Do Nothing) pareto efficient? No, it is pareto dominated by (Preempt; Preempt).

Notes

Deterrence Game

Now imagine that the United States (US) and the European Union (EU) must decide whether to deter a terrorist attack or do nothing.

Let's suppose that deterrence is associated with a cost of 4 for both the deterring country and the other country.

- The deterrer's costs arise from the actual deterrence action that it takes, whereas the non-deterrer's costs arise from now being the terrorists' target of choice.

Notes

Deterrence Game

Each deterrence action provides a private benefit worth 6 (prior to costs being deducted) to the deterring country since it is now safer.

- If only the US (EU) deters, then the US (EU) will get 2 i.e. 6 - 4 and the EU (US) will get -4.
- If the US and EU do nothing, then the net benefits for both players are 0.
- If the US and EU both deter, then each receives a net payoff of -2 i.e. 6 - (2 x 4) as costs of 8 are deduced from private gains of 6.

Notes

Deterrence Game

The setup of the deterrence game is

- Players: $N = \{US, EU\}$
- Actions: $A_i = \{\text{do nothing, deter}\}$ for $i = US,\ EU$.
- Preferences
  - US: (Deter; Do Nothing)>(Do Nothing; Do Nothing)>(Deter; Deter)>(Do Nothing; Deter)
  - EU: (Do Nothing; Deter)>(Do Nothing; Do Nothing)>(Deter; Deter)>(Deter; Do Nothing)

Notes

Deterrence Game

Figure 4.20 Counterterrorism Deterrence Game I

		European Union: Do nothing	European Union: Deter
United States	Do nothing	0, 0	–4, 2
	Deter	2, –4	–2, –2

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy?

Notes

Deterrence Game

Figure 4.20 Counterterrorism Deterrence Game I

		European Union: Do nothing	European Union: Deter
United States	Do nothing	0, 0	–4, 2
	Deter	2, –4	–2, –2

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy? Yes, they both have a dominant strategy to Deter.

What is the NE?

Notes

Deterrence Game

Figure 4.20 Counterterrorism Deterrence Game I

		European Union: Do nothing	European Union: Deter
United States	Do nothing	0, 0	–4, 2
	Deter	2, –4	–2, –2

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy? Yes, they both have a dominant strategy to Deter.

What is the NE? (Deter; Deter)

Is (Deter; Deter) pareto efficient?

Notes

Deterrence Game

Figure 4.20 Counterterrorism Deterrence Game I

		European Union: Do nothing	European Union: Deter
United States	Do nothing	0, 0	–4, 2
	Deter	2, –4	–2, –2

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy? Yes, they both have a dominant strategy to Deter.

What is the NE? (Deter; Deter)

Is (Deter; Deter) pareto efficient? No, it is pareto dominated by (Do Nothing; Do Nothing).

Notes

Preemption-Deterrence Game

Instead of assuming that governments can only implement preemption or deterrence policies, let's now look at a situation where they can implement both types of counterterrorism policy.

The only thing we need to do is determine the payoffs that the countries receive when one preempts and the other deters.

- The deterrer gets a payoff of 6 i.e. $6 + 4 - 4$. In other words, they get 6 from the private benefit associated with the deterrence policy, -4 from the cost of the deterrence policy, and 4 from the public benefit associated with the other country taking a preemptive action.
- The preemptor receives a payoff of -6 i.e. $4 - 6 - 4$. In other words, they get 4 from the public benefit associated with their provision of preemption, -6 from the cost of the preemption policy, and -4 from the deflected costs associated with becoming the target country.

Notes

Preemption-Deterrence Game

The setup of the preemption-deterrence game is

- Players: $N = \{US, EU\}$
- Actions: $A_i = \{$preempt, do nothing, deter$\}$ for $i = US,\ EU$.
- Preferences
  - US: (Deter; Preempt)>(Do Nothing; Preempt)>(Preempt; Preempt)=(Deter; Do Nothing)>(Do Nothing; Do Nothing)>(Preempt; Do Nothing)=(Deter; Deter)>(Do Nothing; Deter)>(Preempt; Deter)
  - EU: (Preempt; Deter)>(Preempts; Do Nothing)>(Preempt; Preempt)=(Do Nothing; Deter)>(Do Nothing; Do Nothing)>(Do Nothing; Preempt)=(Deter; Deter)>(Deter; Do Nothing)>(Deter; Preempt)

Notes

Preemption-Deterrence Game

Figure 4.21 Counterterrorism Deterrence Game II

		European Union		
		Preempt	Do nothing	Deter
United States	Preempt	2, 2	−2, 4	−6, 6
	Do nothing	4, −2	0, 0	−4, 2
	Deter	6, −6	2, −4	−2, −2

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy?

Notes

Preemption-Deterrence Game

Figure 4.21 Counterterrorism Deterrence Game II

		European Union		
		Preempt	Do nothing	Deter
United States	Preempt	2, 2	−2, 4	−6, 6
	Do nothing	4, −2	0, 0	−4, 2
	Deter	6, −6	2, −4	−2, −2

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy? Yes, they both have a dominant strategy to Deter.

What is the NE?

Notes

Preemption-Deterrence Game

Figure 4.21 Counterterrorism Deterrence Game II

		European Union		
		Preempt	Do nothing	Deter
United States	Preempt	2, 2	−2, 4	−6, 6
	Do nothing	4, −2	0, 0	−4, 2
	Deter	6, −6	2, −4	−2, −2

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy? Yes, they both have a dominant strategy to Deter.

What is the NE? (Deter; Deter)

Is (Deter; Deter) pareto efficient?

Notes

Preemption-Deterrence Game

FIGURE 4.21 Counterterrorism Deterrence Game II

		European Union		
		Preempt	Do nothing	Deter
United States	Preempt	2, 2	–2, 4	–6, 6
	Do nothing	4, –2	0, 0	–4, 2
	Deter	6, –6	2, –4	–2, –2

Note: The United States' (the row player's) payoffs are shown first in each cell; the European Union's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell.

Do either player have a dominant strategy? Yes, they both have a dominant strategy to Deter.

What is the NE? (Deter; Deter)

Is (Deter; Deter) pareto efficient? No, it is pareto dominated by (Do Nothing; Do Nothing) *and* (Preempt; Preempt).

Notes

Preemption-Deterrence Game

These games illustrate that states overinvest in counterterrorism deterrence policies and underinvest in counterterrorism preemption policies.

This is not only a theoretical prediction but something that terrorist experts have observed in the real world.

Why do states underinvest in counterterrorism preemption policies?

Notes

Preemption-Deterrence Game

These games illustrate that states overinvest in counterterrorism deterrence policies and underinvest in counterterrorism preemption policies.

This is not only a theoretical prediction but something that terrorist experts have observed in the real world.

Why do states underinvest in counterterrorism preemption policies?

Preemption policies provide public benefits to all potential targets irrespective of whether the targets contribute to the cost of the preemption policies.

In effect, preemption policies are public goods and potential targets like to "free-ride" on the actions of others.

It is your standard collective action or free-rider problem.

Notes

Irrationality of Voting Revisited

An individual's vote is decisive in an election in only two situations: (i) when the election is a tie without her vote and her vote decides the outcome, and (ii) when the election is a one vote win for some candidate without her vote and her vote makes the election a tie.

Given that the likelihood of a voter's vote affecting the outcome of the election is close to zero in most situations, a rational voter will not vote.

It is for this reason that we talk about the **irrationality of voting**.

Remember that this all assumes that voters care about *individually* affecting the outcome of the election.

But what about *groups* of like-minded individuals who wish to affect the election outcome?

Notes

Irrationality of Voting Revisited

Unlike individuals, groups of like-minded individuals may well be decisive in an election.

This helps to explain why party leaders, candidates, trade unions etc. go to particular groups of voters – soccer moms, Nascar dads, African Americans, the religious etc. – using group-based strategies to persuade them to go to the polls as a group.

The implication is that group leaders induce their members to vote (and sometimes not vote) in order to influence the outcome of the election strategically as a group.

Thus, it might be rational to vote instrumentally if one belongs to a particular group that might be decisive in an election.

Notes

Collective Action Problem

But getting voters to act as a group has its own problems – the collective action or free-rider problem.

- The group would benefit if as a collective it acted together; however, each individual in the group has an incentive to free-ride and rely on the actions of others in the group.

Consider a situation in which two voters, Sona and Sean, acting together can change the outcome of the election to their benefit.

- Suppose that their preferred candidate, Gavin, is expected to lose by one vote to his opponent, Rosanne, if neither Sona and Sean vote.
- If only one of them votes, then Gavin and Rosanne are in a tie with each having a 50:50 chance of winning.
- If Gavin wins, Sona and Sean receive 100 units of utility.
- If Rosanne wins, Sona and Sean receive 25 units of utility.
- The cost to voting is 50 units of utility.

Notes

Collective Action Problem

The expected utility of a tied election is

$$EU_{tie} = 0.5(100) + 0.5(25) = 62.5$$

- If both vote, Gavin wins for sure. Sona and Sean receive $100 - 50 = 50$ units of utility.
- If both do not vote, Rosanne wins for sure. Sona and Sean each receive 25 units of utility.
- If only one votes, the election is tied. Sona and Sean receive either $62.5 - 50 = 12.5$ if they voted and 62.5 if they didn't vote.

Notes

Collective Action Problem

		Sean: Vote for Gavin	Sean: No Vote
Sona	Vote for Gavin	50, 50	12.5, 62.5
	No Vote	62.5, 12.5	25, 25

The NE is (No Vote; No Vote).

Sona and Sean will have difficulty acting collectively to elect Gavin even though they both prefer to do so and both would benefit.

Thus, even though the two voters can affect the outcome of the election as a group, each has an incentive to free-ride even though this might lead to a less preferred outcome.

Notes

Private Selective Incentives

Group leaders interested in mobilizing their voters might offer private selective incentives tied to the act of voting.

A **private selective incentive** is a *consumption benefit* that accrues to an individual.

- Private selective incentives include things like food, alcohol, jobs, repairs, transportation, entertainment, raffles, bingo etc..
- Not quite as common today as in the past.

Notes

Private Selective Incentives

Suppose that a group leader provides group members consumption benefits worth 25 units of utility if they vote.

- If both vote, Gavin wins for sure. Sona and Sean receive $100 - 50 + 25 = 75$ units of utility.
- If both do not vote, Rosanne wins for sure. Sona and Sean each receive 25 units of utility.
- If only one votes, the election is tied. Sona and Sean receive either $62.5 - 50 + 25 = 37.5$ if they voted and 62.5 if they didn't vote.

Notes

Private Selective Incentives

		Sean: Vote for Gavin	Sean: No Vote
Sona	Vote for Gavin	75, 75	37.5, 62.5
	No Vote	62.5, 37.5	25, 25

The NE is (Vote for Gavin; Vote for Gavin).

Private selective incentives can help overcome collective action problems. However, they are largely illegal today.

Notes

Social Selective Incentives

Group leaders interested in mobilizing their voters might offer social (and private) selective incentives that are related to their group membership.

A **social selective incentive** is a *collective benefit* that group members obtain from coordinating on the same action.

Suppose that group members care about coordinating on the same action and that if they do, they receive 25 units of utility.

- If both vote, Gavin wins for sure. Sona and Sean receive $100 - 50 + 25 = 75$ units of utility.
- If both do not vote, Rosanne wins for sure. Sona and Sean each receive 25+25=50 units of utility.
- If only one votes, the election is tied. Sona and Sean receive either $62.5 - 50 = 12.5$ if they voted and 62.5 if they didn't vote.

Notes

Social Selective Incentives

		Sean: Vote for Gavin	Sean: No Vote
Sona	Vote for Gavin	75, 75	12.5, 62.5
	No Vote	62.5, 37.5	50, 50

The NE are (Vote for Gavin; Vote for Gavin) and (No Vote; No Vote).

The problem confronting group leaders is to coordinate the group of voters on the voting equilibrium instead of on the non-voting equilibrium.

Social selective incentives are the most common types of selective incentives in established democracies.

Notes

Irrationality of Voting Revisited

In summary, groups use social selective incentives to mobilize voters.

When voters are mobilized by a group, its leaders will choose a mobilization strategy that maximizes their expected utility.

Group leaders will mobilize voters if the following is true:

$$\triangle P_G \times B_G > c_G$$

where $\triangle P_G$ is the effect of mobilizing a group of voters on the election outcome, B_G is the difference in group benefits if the group's preferred candidate wins, and c_G is the cost to the group of mobilization.

As the difference between candidates increases, as elections become closer, and as the costs of voting decrease, turnout goes up.

Notes

Irrationality of Voting Revisited

Individualized instrumental voting cannot by itself explain why voters participate in elections.

Adding in groups helps but requires the assumption that the group can offer group members an incentive to participate.

These incentives are either private or social selective incentives.

Notes

Irrationality of Voting Revisited

There are two types of groups that mobilize voters.

Benefit-Seeking Groups

- Mobilize voters indirectly for office-seeking groups or directly for their own candidates.
- Use their ability to provide voters for office-seeking groups so that the elected officials choose policy positions or provide other collective benefits that please the members of their benefit-seeking group.
- Withhold votes from office-seeking groups if they believe that office-seeking groups are not following their preferences and the cost of mobilization is not worth the return.

Notes

Irrationality of Voting Revisited

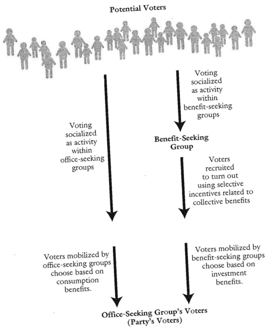
There are two types of groups that mobilize voters.

Office-Seeking Groups

- Use consumption benefits to achieve voter mobilization.
- Will have less reason to respond to the policy preferences of voters since policy – a collective benefit – is not what is being used to mobilize them.
- Voters will base their choices on consumptions benefits since they have no electoral power to induce office seekers to provide them with collective benefits.
- Voters that are mobilized by office-seeking groups may not make the same choices in the voting booth as they would if they were mobilized by benefit-seeking groups.

Notes

Irrationality of Voting Revisited



Notes

What is the State?

The state "is a human community that (successfully) claims the monopoly of the legitimate use of physical force within a given territory" (Max Weber)

"A state is an organization with a comparative advantage in violence, extending over a geographic area whose boundaries are determined by its power to tax constituents." (Douglas North)

States are "relatively centralized, differentiated organizations, the officials of which, more or less, successfully claim control over the chief concentrated means of violence within a population inhabiting a large, contiguous territory." (Charles Tilly)

Notes

What is the State?

Two common factors in all three of these definitions:

1. A given territory.
2. The use of force or the threat of force to control the inhabitants.

A **state** is an entity that uses coercion and the threat of force to rule in a given territory.

Notes

What is the State?

Unlike other social organizations, the state is "a violence producing enterprise." (Lane)

- All states use the threat of force to organize public life.
- States never perfectly monopolize force.
- States never perfectly enforce their will.

Coercion may be justified in different ways, may be used for different purposes, and with different effects. But all states use it.

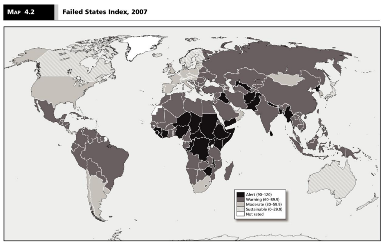
Notes

Failed States

States that cannot coerce are often described as "failed states" – Afghanistan, Somalia, Sierra Leone, Congo, and others.

A **failed state** is a state-like entity that cannot coerce and is unable to successfully control the inhabitants of a given territory.

Figure: Failed States Index



Source: Reproduced with permission from The Fund for Peace.

Notes

Contractarian View of the State

Thought experiment – Hobbes, Locke, Rousseau

- What would life be like without a state? State of Nature.
- How would people behave if they did not need to worry about being punished by a state for killing and stealing?

The **state of nature** is a term used to describe situations in which there is no state.

Hobbes described life in the state of nature as a "war of every man against every man" in which life was "solitary, poor, nasty, brutish, and short."

Notes

Contractarian View of the State

People in the state of nature face a dilemma.

- Given a certain degree of equality in the state of nature, every citizen could gain by attacking his neighbor in a moment of vulnerability.
- The problem is that citizens know that they will frequently be vulnerable themselves.
- Clearly, everyone would be better off if they could all agree not to take advantage of each other.
- But if an act of violence or theft were to happen, it would be better to be the attacker rather than the victim.

Claim: Without a "common power to keep them all in awe," the people will choose to steal and kill.

Notes

State of Nature

Imagine that we have two individuals in the state of nature who have to decide whether or not to steal from each other.

The setup of the **State of Nature Game** is

- Players: $N = \{1, 2\}$
- Actions: $A_i = \{\text{forbear, steal}\}$ for $i = 1, 2$.
- Preferences
  - Player 1: (Steal; Forbear)>(Forbear; Forbear)>(Steal; Steal)>(Forbear; Steal)
  - Player 2: (Forbear; Steal)>(Forbear; Forbear)>(Steal; Steal)>(Steal; Forbear)

Notes

State of Nature

Figure: State of Nature Game

FIGURE 4.6 Solving the State of Nature Game IV

A \ B	Forbear	Steal
Forbear	3,3	1, $\underline{4}$
Steal	$\underline{4}$, 1	$\underline{2}$, $\underline{2}$

Note: Player A's (the row player's) payoffs are shown first in each cell; player B's (the column player's) payoffs are shown second. A comma separates the payoffs for the players in each cell. Payoffs associated with best replies are underlined.

The NE is (Steal; Steal). Steal is a dominant strategy for both players. (Forbear; Forbear) pareto dominates (Steal; Steal) but cannot be sustained as an equilibrium.

Notes

State of Nature

As Hobbes pointed out, individuals will live in a persistent state of fear when there is nobody to keep them in a state of "awe."

The absence of cooperation represents a sort of dilemma – individual rationality leads actors to an outcome that is inferior in the sense that BOTH players agree that the same alternative outcome is better.

"State of nature" may seem abstract, but consider situations in which no single actor can "awe" everyone in society.

- Iraq under U.S. occupation, south central LA or NYC in the 1980s, suburban NJ in the Sopranos, and so on.

In fact, Nobel laureate Robert Fogle argues that Hobbes's state of nature describes most of human history!

Notes

The Social Contract

Hobbes's solution to the state of nature was to create a sovereign with sufficient control of force that individuals would stand in "awe."

He believed that the state of nature was so bad that individuals would be willing to transfer power and so on to the sovereign in exchange for protection.

A **social contract** is an implicit agreement among individuals in the state of nature to create and empower the state. In doing so, it outlines the rights and responsibilities of the state and citizens in regard to each other.

The social contract should produce a sovereign that is strong enough to dole out punishments to individuals who "steal."

- These punishments should be sufficiently large that individuals would no longer have a dominant strategy to "steal."

Notes

Civil Society Game

Figure: Civil Society Game

FIGURE 4.7 Civil Society Game

		B: Forbear	B: Steal
A	Forbear	3, 3	1, 4–p
A	Steal	4–p, 1	2–p, 2–p

Note: p indicates the value of the punishment doled out by the state to anyone who steals.

Same game as before but now with a state lurking in the background punishing individuals who steal.

Notes

Civil Society Game

Is the creation of a state that can dole out punishments sufficient to induce good behavior on the part of the individuals?

Notes

Civil Society Game

Notes

Is the creation of a state that can dole out punishments sufficient to induce good behavior on the part of the individuals?

Figure: Civil Society Game

FIGURE 4.8 Civil Society Game when $p > 1$

		B	
		Forbear	Steal
A	Forbear	$\underline{3}, \underline{3}$	$\underline{1}, 4-p$
	Steal	$4-p, \underline{1}$	$2-p, 2-p$

Note: p = the value of the punishment doled out by the state to anyone who steals. Payoffs associated with best replies are underlined.

Individuals prefer not to steal when $3 > 4 - p$ and $1 > 2 - p$. This happens when $p > 1$.

Civil Society Game

Notes

BUT who is going to be the sovereign and why would he do us all a favor by acting as our policeman?

One common story is that members of civil society are engaged in an exchange relationship with the state. The sovereign agrees to act as a policeman in exchange for taxes that the citizens pay.

Civil Society Game

Notes

BUT who is going to be the sovereign and why would he do us all a favor by acting as our policeman?

One common story is that members of civil society are engaged in an exchange relationship with the state. The sovereign agrees to act as a policeman in exchange for taxes that the citizens pay.

Given that the state will demand tax revenue to carry out its job, it is not immediately obvious that the citizen will choose to leave the state of nature for civil society – much will depend on the tax rate.

So, when is civil society preferred to the state of nature?

Civil Society vs State of Nature

Figure: Civil Society vs State of Nature Game

FIGURE 4.9 **Choosing between the State of Nature and Civil Society**

State of Nature

		B: Forbear	B: Steal
A	Forbear	3, 3	1, $\underline{4}$
A	Steal	$\underline{4}$, 1	$\underline{2}$, $\underline{2}$

Civil Society

		B: Forbear	B: Steal
A	Forbear	$\underline{3-t}$, $\underline{3-t}$	$\underline{1-t}$, $4-t$
A	Steal	$4-p-t$, $\underline{1-t}$	$2-p-t$, $2-p-t$

Note: p = the value of the punishment doled out by the state to anyone who steals; t = the value of the tax imposed by the state. It is assumed that $p > 1$. Payoffs associated with best replies are underlined.

Notes

Civil Society vs State of Nature

The state may be a solution to the state of nature. For this to occur, though, it must be the case that:

1. The punishment imposed by the state for stealing is sufficiently large that individuals prefer to forbear rather than steal.
2. The taxation rate charged by the state for acting as the policeman must not be so large that individuals prefer the state of nature (no state) to civil society (state).

With the particular payoffs we have chosen, this requires that:

- $p > 1$ (punishment must be sufficiently large).
- $t < 1$ (taxation must be sufficiently small).

Notes

Some Thoughts

Political theorists who see the state of nature as particularly dire expect citizens to accept a draconian set of responsibilities in exchange for the "protection" provided by the state.

- Hobbes was writing at the end of a long period of religious war in Europe and civil war in his home country. Thus, he had a firsthand glimpse of what the "war of all against all" looked like and thought that the difference between civil society and the state of nature was effectively infinite.
- It is perhaps for this reason that he believed that almost any level of taxation the state could levy on its citizens in exchange for protection looked like a good deal.

Notes

Some Thoughts

In contrast, political theorists who see civil society as a mere convenience rather than a workable, if inefficient, state of nature, place much greater restrictions on what the state should ask of its citizens.

- From the relative calm of Monticello, Thomas Jefferson – borrowing from social contract theorist John Locke – believed that "life, liberty, and the pursuit of happiness" was possible in the state of nature and that our commitment to the state was so conditional that we should probably engage in revolution every couple of decades.

Contemporary disputes over whether we should reduce civil liberties by giving more power to the state in an attempt to better protect ourselves against terrorist threats directly echo this historical debate between scholars such as Hobbes and Jefferson.

Notes

Some Thoughts

Although the creation of the state may solve the political problem we have with each other, it creates a problem between us and the state.

- If we surrender control over violence to the state, what is to prevent the state from using this power against us?
- "Who will guard the guardian?"

The sovereign. Can't live with him, can't live without him!

Notes

Predatory View of the State

Although the contractarian view of the state focuses on the conflicts of interests between individuals, the predatory view of the state focuses on potential conflicts of interest between citizens and the state.

States are like individuals in the state of nature.

- They face their own security dilemma in the sense that they have potential rivals always vying to take their place.
- The concern for security leads states to use their power to extract resources from others, that is, to predate.

Notes

Predatory View of the State

The sociologist Charles Tilly claims that states resemble a form of organized crime and that they should be viewed as extortion or protection rackets.

As with the contractarian view of the state, the predatory approach sees the state as an organization that trades security for revenue.

BUT, the difference is that the seller of security in the predatory approach happens to represent the key threat to the buyer's continued security.

The Mafia vs The British Army ▸ here

Notes

Predatory View of the State

"War makes the state . . . States make war."

The need to compete with external rivals creates the pressure for rulers to raise revenues to fight wars.

The need to extract a lot of revenues poses a problem for rulers.

- One solution to this problem is to eliminate internal rivals.
- The elimination of internal rivals and the development of the capacity to extract resources is the process of state making.

State formation is not the intent of rulers, but the result. Rulers are just trying to grasp power.

Notes

Predatory View of the State

War making: Eliminating or neutralizing their own rivals outside the territories in which they have clear and continuous priority as wielders of force.

State making: Eliminating or neutralizing their rivals inside those territories.

Protection: Eliminating or neutralizing the enemies of their clients.

Extraction: Acquiring the means of carrying out the first three activities.

"Power holders' pursuit of war involved them (the state) willy-nilly in extraction of resources for war making from the populations over which they had control and in the promotion of capital accumulation by those who could help them borrow and buy. War making, extraction, and capital accumulation interacted to shape European State making. Power holders did not undertake those three momentous tasks with the intention of creating national states . . . Nor did they ordinarily foresee that national states would emerge from war making, extraction and capital accumulation." (Charles Tilly).

Notes

Predatory View of the State

" . . . instead, they warred in order to check or overcome their competitors and thus to enjoy the advantages of power within a secure or expanding territory. To make more effective war, they attempted to locate more capital. In the short run, they might acquire that capital by conquest, by selling off their assets, or by coercing or dispossessing accumulators of capital. In the long run, the quest inevitably involved them in establishing regular access to capitalists who could supply and arrange credit and in imposing one form of regular taxation or another on the people and activities within their spheres of control."

The modern state arose as a by-product of the attempts of leaders to survive.

Notes

Predatory View of the State

The act of extraction "entailed the elimination, neutralization, or cooptation of the great Lord's [internal] rivals; thus it led to state-making. As a by-product, it created organization in the form of tax-collection agencies, police forces, courts, exchequers, account keepers; thus, it again led to state making. To a lesser extent, war making likewise led to state making through the expansion of the military organization itself, as a standing army, war industries, supporting bureaucracies, and (rather later) schools grew up within the state apparatus. All of these structures checked potential rivals and opponents."

War makes states!

Notes

Limits to State Predation

The need to extract resources from their clients often placed constraints on the predation of early modern leaders.

- Don't want to tax too much because this inhibits investment.
- If you don't predate too much, then you might be able to benefit from voluntary compliance.
- By regulating their predatory instincts, rulers could opt to increase their net extractive capacity by reducing the costs of conducting business and by taking a smaller portion of a larger pie.

Obviously, not all states were successful in limiting their predation in this way, and as a result, the character and consequences of rule exhibited quite a variety across early modern Europe.

We'll come back to the question of why some states limit their predation and others don't a little later.

Notes

An Aside: *Repeated* State of Nature Game

Can cooperation occur in the state of nature without the state?

We saw that (Forbear; Forbear) was not possible as a NE in the state of nature in a one-shot game.

But, what if the players repeatedly interacted with each other?

Notes

An Aside: *Repeated* State of Nature Game

Can cooperation occur in the state of nature without the state?

We saw that (Forbear; Forbear) was not possible as a NE in the state of nature in a one-shot game.

But, what if the players repeatedly interacted with each other?

To answer this, we'll take a (very brief) look at **repeated games**.

It turns out that cooperation is possible in the state of nature if the State of Nature Game is infinitely repeated.

In a repeated game, each player conditions her action at each point in time on the other players' previous actions.

Notes

An Aside: *Repeated* State of Nature Game

The outcome of a repeated game is a sequence of outcomes of a strategic game.

Each player associates a payoff with each outcome of the strategic game and evaluates each sequence of outcomes in the repeated game by the discounted sum (or present value) of the associated sequence of payoffs.

More precisely, each player i has a payoff function u_i for the strategic game and a discount factor δ_i between 0 and 1 such that she evaluates the sequence $(a^1, a^2, \ldots, a^t)$ of outcomes of the strategic game by the sum

$$u_i(a^1) + \delta_i u_i(a^2) + \delta_i^2 u_i(a^3) \ldots + \delta_i^{T-1} u_i(a^T) = \sum_{t=1}^{T} \delta_i^{t-1} u_i(a^t)$$

where a^t indicates the action profile chosen in period t and δ_i^t is the discount factor of player i raised to the power t.

Notes

Discount Factor

Discount Factor (δ).

- This tells us the rate at which future benefits are discounted compared with today's benefits.
- Essentially, it tells us how much people care about the future.
- Discount factor (δ) is bounded, that is, $0 < \delta < 1$.

Example: $1,000 today or in a month's time.

- If it did not matter to you whether you got the money today or in a month's time, your discount factor would be close to 1.
- If you really wanted to get the money today, your discount factor would be close to 0.

Notes

Present Value or Discounted Sum

Say something is worth 5 in the first period. It will be worth 5δ in the second period, $5\delta^2$ in the third period, $5\delta^3$ in the third period, and so on.

So, the **present value** or **discounted sum** of this good is

$$5 + 5\delta + 5\delta^2 + 5\delta^3 + 5\delta^4 + \ldots$$

A useful fact:

$$1 + \delta + \delta^2 + \delta^3 + \delta^4 + \ldots = \frac{1}{1-\delta}$$

So, the present value or discounted sum of the good is $\frac{5}{1-\delta}$.

Notes

An Aside: *Repeated* State of Nature Game

Now that we know what a discount factor is and how to calculate the present value of a future stream of benefits, we can examine what happens when we repeatedly play the State of Nature Game.

How will the players play the game now?

Notes

An Aside: *Repeated* State of Nature Game

Now that we know what a discount factor is and how to calculate the present value of a future stream of benefits, we can examine what happens when we repeatedly play the State of Nature Game.

How will the players play the game now?

One strategy that they might use is called a **grim trigger strategy**:

- Choose Forbear as long as the other player chooses Forbear
- If in any period the other player chooses Steal, then choose Steal in every subsequent period

This strategy begins by playing cooperatively and continues doing so until the other player defects; a single defection by the opponent triggers relentless ("grim") defection, which may be interpreted as retaliatory "punishment."

Notes

An Aside: *Repeated* State of Nature Game

Figure: Civil Society vs State of Nature Game

FIGURE 4.23 State of Nature Game Revisited

		B: Forbear	B: Steal
A	Forbear	3, 3	1, 4
A	Steal	4, 1	2, 2

The present value of choosing Forbear is

$$3 + 3\delta + 3\delta^2 + 3\delta^3 + 3\delta^4 \ldots = \frac{3}{1-\delta}$$

The present value of choosing Steal is

$$\begin{aligned} 4 + 2\delta + 2\delta^2 + 2\delta^3 + 2\delta^4 \ldots &= 4 + 2\delta(1 + \delta + \delta^2 + \delta^3 + \delta^4) \\ &= 4 + 2\delta\left(\frac{1}{1-\delta}\right) = 4 + \frac{2\delta}{1-\delta} \end{aligned}$$

Notes

An Aside: *Repeated* State of Nature Game

Individuals in the state of nature will prefer to forbear rather than steal when the present value of forbear is greater than the present value of steal.

$$\begin{aligned} \frac{3}{1-\delta} &\geq 4 + \frac{2\delta}{1-\delta} \\ \frac{3}{1-\delta} &\geq \frac{4-4\delta}{1-\delta} + \frac{2\delta}{1-\delta} \\ \frac{3}{1-\delta} &\geq \frac{4-4\delta+2\delta}{1-\delta} \\ 3 &\geq 4 - 2\delta \\ 2\delta &\geq 1 \\ \delta &\geq \frac{1}{2} \end{aligned}$$

If $\delta \geq 0.5$, given our payoffs, then individuals in the state of nature using grim trigger strategies will choose to forbear rather than steal.

Notes

An Aside: *Repeated* State of Nature Game

(Forbear; Forbear) can be sustained as an equilibrium using a grim trigger strategy as long as individuals are sufficiently concerned about the potential benefits of future cooperation (δ is sufficiently large) and the game is *infinitely repeated*.

Thus, the state is not strictly necessary to achieve cooperation.

Recall that the State of Nature game is just the Prisoner's Dilemma.

Our brief analysis of repeated games helps to explain why things like trade, environmental, and arms agreements can be achieved and sustained much more easily between states that frequently interact with each other.

Even without a world court to enforce these types of agreements, states might agree to cooperate if they value the potential benefits of future cooperation sufficiently highly.

Notes

An Aside: *Repeated* State of Nature Game

Note that cooperation **cannot** be sustained if the game is only *finitely repeated*. Why?

Notes

An Aside: *Repeated* State of Nature Game

Note that cooperation **cannot** be sustained if the game is only *finitely repeated*. Why?

In the last period T, each player has a dominant strategy to Steal and have no future periods to punish her.

Given that you know your opponent is going to steal in the last period, both players' best action in period $T-1$ is also to steal.

Given that you know your opponent is going to steal in the $T-1$ period, both players' best action in period $T-2$ is also to steal.

This unravels all the way back such that both players choose Steal from the first period.

Notes

An Aside: *Repeated* State of Nature Game

This result from our infinitely-repeated State of Nature game runs directly counter to the claims of social contract theorists like Hobbes and provides support for groups like anarchists who believe that society can survive, and thrive, without a state.

But relying on cooperation to come about through a decentralized process without the state may not be the best thing to do.

1. (Steal; Steal) can also be sustained as an equilibrium by the grim trigger strategy.
2. It is costly for individuals to cooperate without a state because individuals have to monitor each other's behavior and be willing to punish noncompliance.

Notes

Electoral Competition

What factors determine the number of political parties and the policies they propose?

How is the outcome of an election affected by the electoral system and the voters' preferences over policies?

We're going to look at a foundational model called Hotelling's (or Downs') model of electoral competition.

The model has two stages:

1. Electoral competition where candidates choose their policy positions.
2. Elections where citizens vote for candidates.

Notes

Electoral Competition

Stage 1: Electoral Competition

- **Players:** Two candidates $\{A, B\}$
- **Actions:** The candidates simultaneously announce their "policy positions" p_j i.e. a real number on a one-dimensional policy space given by the set [0,100]. Both policy positions are made public to the electorate.
- **Preferences:** Each candidate prefers to win than to tie (in which case we assume that the winner is determined by a coin toss) and to tie than to lose. No candidate has an ideological attachment to any policy position – they are pure office-seekers.

Notes

Electoral Competition

Stage 2: Elections

The election is conducted by simple majority (plurality) rule – the candidate with the most votes wins.

- **Players:** An odd number of voters.
- **Actions:** Each voter's set of actions consists of "voting for A" or "voting for B" i.e. $\{A, B\}$.
- **Preferences:** Voters have single-peaked preferences indicating their ideal policy position x_i i.e. a real number on a one-dimensional policy space given by the set [0,100]. Voter i receives his highest utility (payoff) if the winning policy position p^w is equal to her ideal point x_i. The further away p^w is from her ideal position x_i, the lower is her payoff. We can represent such preferences by the following utility function, $u_i(x_i, p^w) = -|x_i - p^w|$.

The ideal positions of voters are given, but the policy positions of the two candidates are determined as an equilibrium in the first stage of the model.

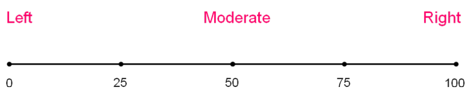
Notes

Policy Space

Assumptions

- The policy space is one-dimensional – we can think of the left-right policy space.
- Each candidate can choose any policy position p_j from the set [0,100].

Figure: Policy Space

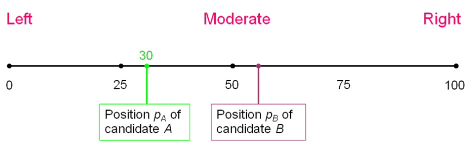


Notes

Candidate Positions

Candidate A might choose policy position $p_A = 30$ and candidate B might choose policy position $p_B = 55$.

Figure: Candidate Positions

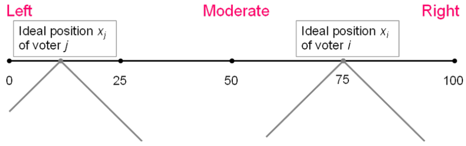


Notes

Voter Ideal Points

Each voter i has an ideal point x_i, which can be any real number from the set [0,100].

Figure: Voter Ideal Points



Notes

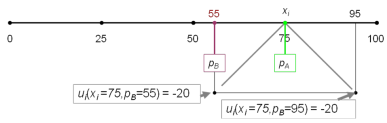
Voter Utility Functions

A voter's utility function is $u_i(x_i, p_j) = -|x_i - p_j|$.

Suppose that the voter's ideal point is 75 and that there are two candidates, $A = 75$ and $B = 55$.

- $u_i(x_i, p_A) = -|x_i - p_A| = -|75 - 75| = 0$
- $u_i(x_i, p_B) = -|x_i - p_B| = -|75 - 55| = -20$
- Voter i prefers A to B because $u_i(x_i, p_A) = 0 > u_i(x_i, p_B) = -20$.

Figure: Voter Utility Functions



Notes

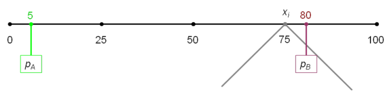
Voter Utility Functions

A voter's utility function is $u_i(x_i, p_j) = -|x_i - p_j|$.

Suppose that the voter's ideal point is 75 and that there are two candidates, $A = 5$ and $B = 80$.

- $u_i(x_i, p_A) = -|x_i - p_A| = -|75 - 5| = -70$
- $u_i(x_i, p_B) = -|x_i - p_B| = -|80 - 55| = -5$
- Voter i prefers B to A because $u_i(x_i, p_B) = -5 > u_i(x_i, p_A) = -70$.

Figure: Voter Utility Functions



Basically, voters prefer candidates that are located closer to their ideal points.

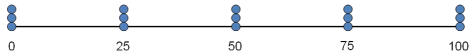
Notes

Equilibrium Elections

Assume the following concrete distribution of voter preferences: there are three voters with ideal points at each of the positions 0, 25, 50, 75, and 100.

What are the Nash equilibria of the election? Consider only equilibria in weakly dominant actions.

Figure: Example

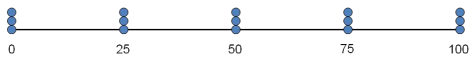


Notes

Equilibrium Elections

In the only Nash equilibrium in weakly dominant actions of the election stage, each voter i votes for her preferred candidate i.e. the candidate whose policy position is closer to her own ideal point. And she votes randomly for one of the two candidates – she tosses a coin – if both candidate policy positions are equally close to her ideal point (she is indifferent between the candidates in this case).

Figure: Example



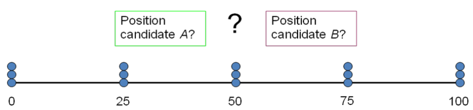
Notes

Equilibrium Policy Positions

The candidates anticipate the Nash equilibrium from the election stage – backward induction, something we'll look at when we learn about extensive form games.

What are the equilibrium policy positions chosen by both candidates?

Figure: Example



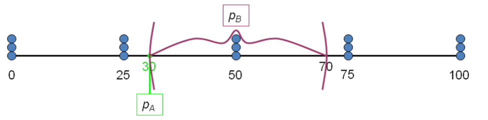
Notes

Equilibrium Policy Positions

Assume candidate A chooses a policy position to the left of position 50, e.g. she chooses $p_A = 30$.

Can this be an equilibrium?

Figure: Example



Notes

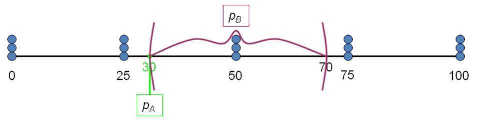
Equilibrium Policy Positions

Assume candidate A chooses a policy position to the left of position 50, e.g. she chooses $p_A = 30$.

Can this be an equilibrium?

No, because candidate B can win by choosing any policy position for which $30 < p_B < 70$ holds. In this way she will receive 9 votes whereas candidate A will only receive 6 votes.

Figure: Example



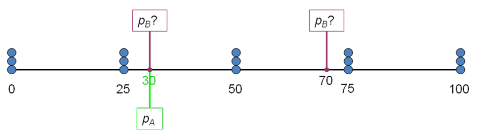
Notes

Equilibrium Policy Positions

If B chooses $p_B = 30$ or $p_B = 70$, each candidate has an equal chance of winning the election.

But if that is the case, then B prefers choosing $30 < p_B < 70$ because that will yield victory for sure.

Figure: Example



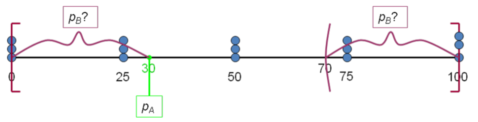
Notes

Equilibrium Policy Positions

If B chooses $p_B < 30$ or $p_B > 70$, candidate A will receive at least 9 votes and will win the election.

But if that is the case, then B prefers choosing $30 < p_B < 70$ because that will yield victory for sure.

Figure: Example



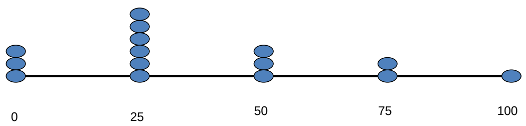
Notes

Equilibrium Policy Positions

Now let's now look at the following distribution of voter preferences: there are 3 voters with ideal positions at position 0, 6 at position 25, 3 at position 50, 2 at position 75, and 1 at position 100.

What are the Nash equilibrium policy positions of candidate A and candidate B?

Figure: Example 2



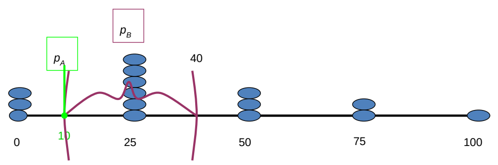
Notes

Equilibrium Policy Positions

Assume candidate A chooses a policy position to the left of position 25, e.g. she chooses $p_A = 10$.

Can this be an equilibrium?

Figure: Example 2



Notes

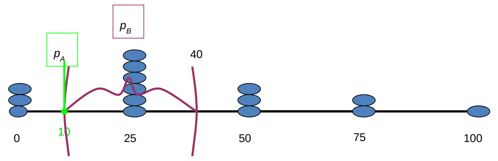
Equilibrium Policy Positions

Assume candidate A chooses a policy position to the left of position 25, e.g. she chooses $p_A = 10$.

Can this be an equilibrium?

Candidate B can win by choosing any policy position for which $10 < p_B < 40$ holds. In this way she will receive 12 votes whereas candidate A will only receive 3 votes.

Figure: Example 2



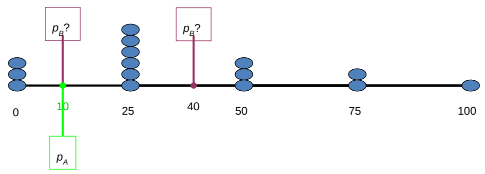
Notes

Equilibrium Policy Positions

If B chooses $p_B = 10$, each candidate has an equal chance of 50% of winning the election. If B chooses $p_B = 40$, she has 89.0625% chance (why?) of winning the election.

But B will prefer choosing $10 < p_B < 40$ because this will yield her a certain (100%) victory.

Figure: Example 2



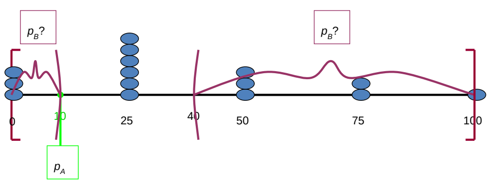
Notes

Equilibrium Policy Positions

If B chooses $p_B < 10$, candidate A will receive 12 votes and will win the election. And, if B chooses $p_B > 40$, candidate A will receive at least 9 votes and will win the election.

But then B prefers choosing $10 < p_B < 40$ because this will yield her a victory.

Figure: Example 2



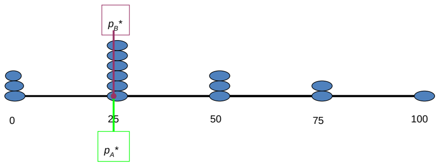
Notes

Equilibrium Policy Positions

Candidate A realizes that she will surely lose if she chooses $p_A = 10$ (or, in fact, any other $p_A \neq 25$).

Is ($p_A = 25$, $p_B = 25$) an equilibrium?

Figure: Example 2



Notes

Equilibrium Policy Positions

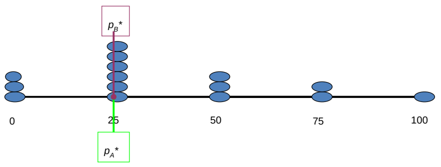
Candidate A realizes that she will surely lose if she chooses $p_A = 10$ (or, in fact, any other $p_A \neq 25$).

Is ($p_A = 25$, $p_B = 25$) an equilibrium?

In this case both candidates have an equal chance (50%) of winning the election.

Yes, this is indeed an equilibrium, because given the other candidate chooses $p_{-j} = 25$, deviating from $p_j = 25$ to the left or right results in a defeat of candidate j (because $-j$ will receive at least 9 votes).

Figure: Example 2



Notes

Electoral Competition

After analyzing the two different distributions of voter preferences, we can see that the resulting Nash equilibria have similarities that may be due to a more general structure.

A position that turns out to have special significance is the ideal point of the median voter:

- The median voter's position is the position m with the property that exactly half of the voters' ideal positions are at most, and half of the voters' ideal positions are at least m.
- For our first distribution of voter preferences m was equal to 50 and in our second distribution we had $m = 25$.

Notes

Electoral Competition

The distribution of voters' ideal positions over the set of all possible positions is arbitrary.

- In particular, this distribution need not be uniform: a large fraction of the voters may have ideal points close to one position, while few voters have ideal points close to some other position.
- Moreover, the distribution of voters' ideal points may be discrete or continuous.

Notes

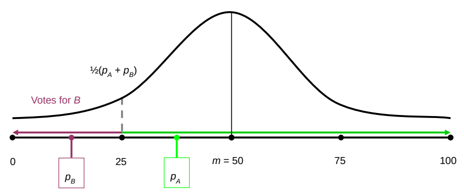
Continuous Voter Distribution

Let's consider the following continuous distribution of voter preferences.

Assume candidate A chooses any policy position left from the median, or $p_A < m$.

What happens if candidate B chooses $p_B < p_A$?

Figure: Example 3



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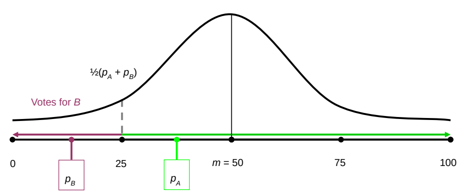
What happens if candidate B chooses $p_B < p_A$? Candidate B will lose!

What happens if candidate B chooses $p_B = p_A$?

Figure: Example 3



Notes

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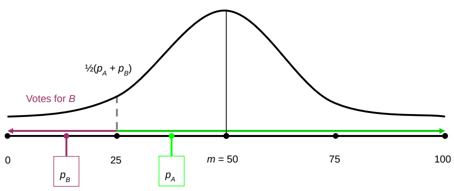
Assume candidate A chooses any policy position left from the median, or $p_A < m$.

What happens if candidate B chooses $p_B < p_A$? Candidate B will lose!

What happens if candidate B chooses $p_B = p_A$? The election results in a 50% chance for each candidate!

Figure: Example 3

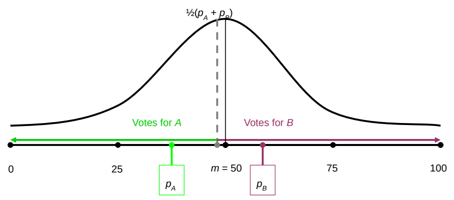


Notes

Continuous Voter Distribution

What happens if candidate B chooses $p_B > p_A$?

Figure: Example 3



Notes

Continuous Voter Distribution

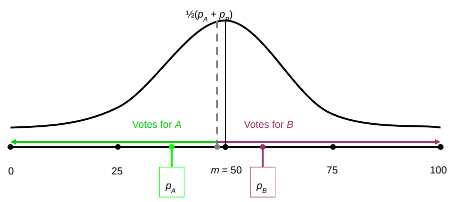
What happens if candidate B chooses $p_B > p_A$?

Candidate B will win as long as the dividing line between her supporters and those of candidate A is less than m (which is the case in our graph below).

If the dividing line lies to the right of m, then candidate B loses.

Hence, in order to win, candidate B should chose a policy position such that $\frac{1}{2}(p_A + p_B) < m$ or $p_B < 2m - p_A$.

Figure: Example 3



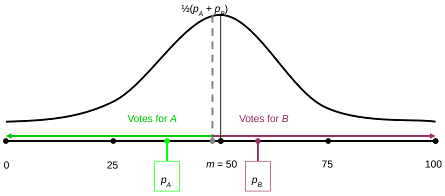
Notes

Continuous Voter Distribution

Hence, in order to win, candidate B should chose a policy position such that $\frac{1}{2}(p_A + p_B) < m$ or $p_B < 2m - p_A$.

Note that if candidate B chooses her policy position such that $\frac{1}{2}(p_A + p_B) = m$, both candidates receive the same number of votes, which results in a 50% chance of winning for each.

Figure: Example 3



Notes

Continuous Voter Distribution

From the previous slides it follows that the best responses of candidate B to $p_A < m$ are characterized by

$$p_A < p_B < 2m - p_A$$

A symmetric argument applies to the case in which $p_A > m$. In this case, the best responses of candidate B are characterized by

$$2m - p_A < p_B < p_A$$

Finally consider the case in which $p_A = m$. In this case candidate B's single best response is to choose the same position, m. If B chooses any other position, then she loses, whereas if she chooses m, then the election ends up in a 50% chance of winning for each candidate.

The best responses of candidate A to all possible policy positions of candidate B are derived in the same way.

Notes

Median Voter Theorem

In simple majority (plurality) elections,

- if the voters' ideal points (i.e., voters policy preferences) can be represented by points along a single dimension,
- if all voters vote deterministically for the candidate that commits to a policy position closest to their own ideal point,
- if there are only two candidates,

then if the candidates want to maximize their chance of winning they will both commit to the policy position preferred by the median voter.

This is the unique Nash equilibrium of the game.

Notes

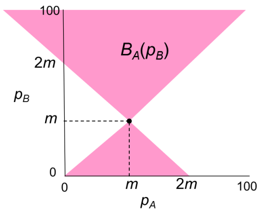
Electoral Competition: Best Response Functions

Candidate A's best response function is defined by

$$B_A(p_B) = \begin{cases} \{p_A : p_B < P_A < 2m - p_B\} & \text{if } p_B < m \\ \{m\} & \text{if } p_B = m \\ \{p_A : 2m - p_B < p_A < p_B\} & \text{if } p_B > m \end{cases}$$

The pink area and the black point show the best responses of candidate A to all possible policy positions of candidate B.

Figure: Best Response Functions



Notes

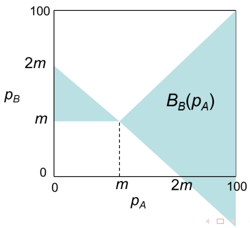
Electoral Competition: Best Response Functions

Candidate B's best response function is defined by

$$B_B(p_A) = \begin{cases} \{p_B : p_A < P_B < 2m - p_A\} & \text{if } p_A < m \\ \{m\} & \text{if } p_A = m \\ \{p_B : 2m - p_A < p_B < p_A\} & \text{if } p_A > m \end{cases}$$

The blue area and the black point show the best responses of candidate B to all possible policy positions of candidate A.

Figure: Best Response Functions



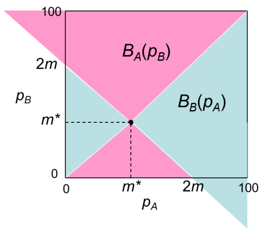
Notes

Electoral Competition: Mutual Best Responses

Putting both graphs of best responses into one graph reveals that there is only one point of mutually best responses: $(p_A = m^*, p_B = m^*)$. Hence, we have a unique Nash equilibrium.

Note that the white lines or borders between both candidates' areas of best responses, are not best responses.

Figure: Mutual Best Responses



Notes

Electoral Competition: Examining Action Profiles

We can also make a direct argument that (m^*, m^*) is the unique Nash equilibrium of the game, without constructing the best response functions.

Notes

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First, (m, m) is an equilibrium: it results in a 50% chance of winning for each candidate, and if either candidate chooses a position different from m, then she loses.

Notes

Electoral Competition: Examining Action Profiles

We can also make a direct argument that (m^*, m^*) is the unique Nash equilibrium of the game, without constructing the best response functions.

First, (m, m) is an equilibrium: it results in a 50% chance of winning for each candidate, and if either candidate chooses a position different from m, then she loses.

Second, no other pair of policy positions is a Nash equilibrium, by the following argument:

- If one candidate loses, then she can do better by moving to m, where she either wins outright (if her opponent's position is different from m) or ties for first place (if her opponent's positions is m).
- If the candidates tie in expectation (because their positions are either the same or symmetric about m), then either candidate can do better by moving to m, where she wins outright.

Notes

War of Attrition

War of Attrition game

The game was originally posed as a model of a conflict between two animals fighting over prey:

- Each animal chooses the time at which it intends to give up.
- When an animal gives up, its opponent obtains all the prey (and the time at which the winner intended to give up is irrelevant).
- If both animals give up at the same time, then each has an equal chance of obtaining the prey.
- Fighting is costly: each animal prefers as short a fight as possible.

The game could be used to model any situation where the "prey" is some indivisible object, and "fighting" is any costly action.

Notes

War of Attrition

To define the game precisely, let time be a continuous variable that starts at 0 and runs indefinitely.

Assume that the value party i attaches to the object in dispute is $v_i > 0$ and the value she attaches to a 50% chance of obtaining the object is $\frac{v_i}{2}$.

Each unit of time that passes before the dispute is settled (i.e. one of the parties concedes) costs each party one unit of payoff.

- Thus, if player i concedes first, at time t_i, her payoff is $-t_i$ (she spends t_i units of time and does not obtain the object).

If the other player concedes first, at time t_{-i}, player i's payoff is $v_i - t_{-i}$ (she obtains the object after t_{-i} units of time).

If both players concede at the same time, player i's payoff is $\frac{1}{2}v_i - t_i$, where t_i is the common concession time.

Notes

War of Attrition

The setup of the War of Attrition game:

- **Players:** The two parties to a dispute.
- **Actions:** Each player's set of actions is the set of possible concession times (non-negative numbers).
- **Preferences:** Player i's preferences are represented by the payoff function

$$u_i(t_1, t_2) = \begin{cases} -t_i & \text{if } t_i < t_{-i} \\ \frac{1}{2}v_i - t_i & \text{if } t_i = t_{-i} \\ v_i - t_{-i} & \text{if } t_i > t_{-i} \end{cases}$$

  where $-i$ is the other player.

Notes

War of Attrition

To make the ideas precise, we can study player i's payoff function for various fixed values of t_{-i}, the concession time of player $-i$.

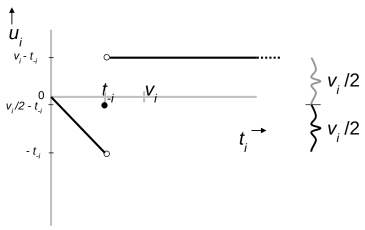
The three cases that the intuitive argument suggests are qualitatively different are shown in the following figures.

Notes

War of Attrition

Case 1: $t_{-i} < v_i$

Figure: War of Attrition I



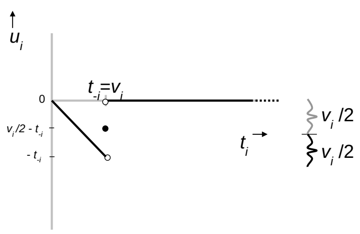
Player i's best response is her action for which her payoff is highest: the set of times after t_{-i} if $t_{-i} < v_i$.

Notes

War of Attrition

Case 2: $t_{-i} = v_i$

Figure: War of Attrition II



Player i's best response is her action for which her payoff is highest: 0 and the set of times after t_{-i} if $t_{-i} < v_i$.

Notes

War of Attrition

Case 3: $t_{-i} > v_i$

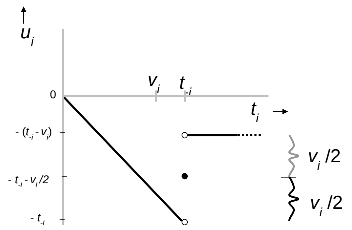


Figure: War of Attrition III

Player i's best response is her action for which her payoff is highest: 0 if $t_{-i} > v_i$.

Notes

War of Attrition

In summary, player i's best response function is defined by

$$B_i(t_{-i}) = \begin{cases} \{t_i : t_i > t_{-i}\} & \text{if } t_{-i} < v_i \\ \{t_i : t_i = 0 \text{ or } t_i > t_{-i}\} & \text{if } t_{-i} = v_i \\ \{0\} & \text{if } t_{-i} > v_i \end{cases}$$

Notes

War of Attrition

For the case in which $v_1 > v_2$, the best response function for player 1 is shown below.

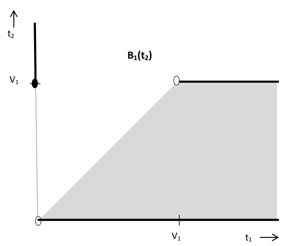


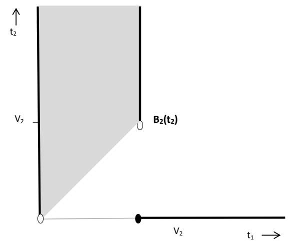
Figure: Best Response Function

Notes

War of Attrition

For the case in which $v_1 > v_2$, the best response function for player 2 is shown below.

Figure: Best Response Function



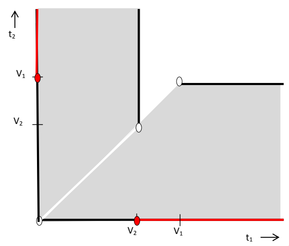
Notes

War of Attrition

Superimposing the players' best response functions, we see that there are two areas of intersection: the vertical axis at and above v_1 and the horizontal axis at and to the right of v_2. Thus (t_1, t_2) is a Nash equilibrium of the game if and only either

$$t_1 = 0 \text{ and } t_2 \geq v_1 \quad \text{or} \quad t_2 = 0 \text{ and } t_1 \geq v_2$$

Figure: Best Response Functions

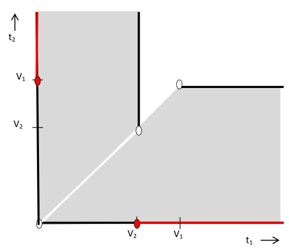


Notes

War of Attrition

In words, in every Nash equilibrium either player 1 concedes immediately and player 2 concedes at time v_1 or later, or player 2 concedes immediately and player 1 concedes at time v_2 or later.

Figure: Best Response Functions



Notes

War of Attrition

The War of Attrition is an example of a "game of timing", in which each player's payoff depends sensitively on whether her action is greater or less than the other player's action (i.e., time chosen).

In many such games, each player's strategic variable is the time at which to act, hence the name "game of timing".

Notes

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