

Problems with Group Decision Making

There are two ways of evaluating political systems:

1. Do institutions produce good outcomes?
2. Are the institutions good, fair, or just irrespective of the outcomes they produce?

Does the **process** by which democratic governments make decisions for everyone have appealing properties that make it morally or normatively attractive above and beyond any material benefits it might produce?

Many people like democracy because they believe it to be a fair way to make decisions.

One commonsense notion of fairness is that group decisions should reflect the preferences of the **majority** of group members.

Most people probably agree that a fair way to decide between two options is to choose the option that's preferred by the most people.

At its heart, democracy is a system in which the majority rules.

Majority rule, though, is often a lot more complicated and less fair than our commonsense intuition about it might suggest.

Social choice theory addresses the voting procedures that govern and describe how individual preferences are aggregated to form a collective group preference.

Majority Rule and Condorcet's Paradox

An actor is **rational** if they possess a **complete** and **transitive** preference ordering over a set of outcomes.

An actor has a **complete preference ordering** if they can compare each pair of elements (call them x and y) in a set of outcomes in one of the following ways - either the actor prefers x to y , y to x , or they're indifferent between them.

An actor has a **transitive preference ordering** if for any x , y , and z in the set of outcomes, it's the case that if x is weakly preferred to y , and y is weakly preferred to z , then it must be the case that x is weakly preferred to z .

Condorcet's paradox illustrates that a group composed of individuals with rational preferences doesn't necessarily have rational preferences as a collective.

Individual rationality isn't sufficient to ensure group rationality.

Imagine a city council made up of three individuals that must decide whether to:

1. Increase social services (I)
2. Decrease social services (D)
3. Maintain current levels of services (C)

City Council Preferences for the Level of Social Service Provision

| Left-wing councillors | Centrist councillors | Right-wing councillors |
|-----------------------|----------------------|------------------------|
| $I > C > D$ | $C > D > I$ | $D > I > C$ |

Let's suppose the council employs majority rule to make its group decision.

One possibility is a round-robin tournament.

A **round-robin tournament** pits each competing alternative against every other alternative an equal number of times in a series of pair-wise votes.

Outcomes from the Round-Robin Tournament

| Round | Contest | Winner | Majority that produced victory |
|-------|-----------------------|----------|--------------------------------|
| 1 | Increase vs. decrease | <i>D</i> | Centrist and right |
| 2 | Current vs. increase | <i>I</i> | Left and right |
| 3 | Current vs. decrease | <i>C</i> | Left and centrist |

The group can't decide! Each alternative wins one round.

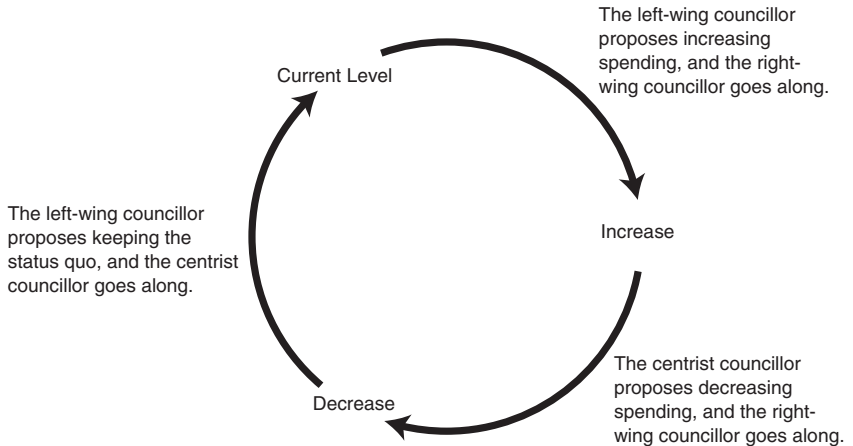
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A group of rational individuals is incapable of making a rational decision for the group as a whole.

There's no 'majority' to speak of – a different majority supports the winning alternative or outcome in each round.

An Example of Cyclical Majorities



Our example demonstrates how a set of rational individuals can form a group with intransitive preferences.

In the real world, though, we see deliberative bodies make decisions all the time and they don't appear to be stuck in an endless cycle.

Why?

There are two broad reasons:

1. Preference orderings.
2. Decision-making rules.

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The councillors having a particular set of preference orderings.

Suppose the right-wing councillor's preferences are instead a mirror image of the left-wing councillor's.

Their preferences are now $D > C > I$ instead of $D > I > C$.

If the right-wing councillor's preferences are $D > C > I$, then C is a **Condorcet winner**.

An option is a **Condorcet winner** if it beats all the other options in a series of pair-wise contests.

Majority rule isn't necessarily incompatible with rational group preferences.

Condorcet's Paradox only shows that it's possible for a group of individuals with transitive preferences to produce a group that behaves as if it has intransitive preferences.

How often are individuals likely to hold preferences that cause intransitivity?

Proportion of Possible Strict Preference Orderings without a Condorcet Winner

| | Number of voters | | | | | | |
|------------------------|------------------|-------|-------|-------|-------|---|----------|
| Number of alternatives | 3 | 5 | 7 | 9 | 11 | → | Limit |
| 3 | 0.056 | 0.069 | 0.075 | 0.078 | 0.080 | | 0.088 |
| 4 | 0.111 | 0.139 | 0.150 | 0.156 | 0.160 | | 0.176 |
| 5 | 0.160 | 0.200 | 0.215 | | | | 0.251 |
| 6 | 0.202 | | | | | | 0.315 |
| ↓ | | | | | | | ↓ |
| Limit | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | | 1.000 |

In general, we can't rely on majority rule to produce a coherent sense of what the group wants, especially if there are no institutional mechanisms for keeping the number of voters small or weeding out some of the alternatives.

Many political decisions involve bargaining and hence an infinite number of alternatives!

Condorcet's Paradox indicates that restricting group decision making to sets of rational individuals is no guarantee the group as a whole will exhibit rational tendencies.

Group intransitivity is unlikely when the set of feasible options is small, but is almost certain when the set of feasible alternatives gets large.

As a result, it's impossible to say that the majority 'decides' except in very restricted circumstances.

The analytical insight from Condorcet's Paradox suggests that group intransitivity should be common.

But we observe a surprising amount of stability in group decision making in the real world.

Perhaps this has something to do with the decision-making rules that we use.

1. Preference orderings.
2. Decision-making rules.

The Borda Count and the Reversal Paradox

The **Borda count** asks individuals to rank potential alternatives from their most to least preferred and then assign points to reflect this ranking.

The alternative with the most 'points' wins.

Determining the Level of Social Service Provision using the Borda Count

| Points awarded | | | | |
|-------------------|-----------|----------|------------|-------------------|
| Alternative | Left-wing | Centrist | Right-wing | Borda count total |
| Increase spending | 3 | 1 | 2 | 6 |
| Decrease spending | 1 | 2 | 3 | 6 |
| Current spending | 2 | 3 | 1 | 6 |

Using the same preferences as before, the Borda count doesn't provide a clear winner either.

A more troubling aspect of this decision rule can be seen if we consider the introduction of a fourth alternative, future cuts (FC).

| Left-wing | Centrist | Right-wing |
|------------------|------------------|------------------|
| $I > C > D > FC$ | $C > D > FC > I$ | $D > FC > I > C$ |

Note: I = an increase in social service provision; D = a decrease in social service provision; C = a maintenance of current levels of social service provision; FC = future cuts in social service provision; $>$ means "is strictly preferred to."

| | Points awarded | | | |
|-------------------------|----------------|----------|------------|-------------------|
| Alternative | Left-wing | Centrist | Right-wing | Borda count total |
| Increase spending | 3 | 0 | 1 | 4 |
| Decrease spending | 1 | 2 | 3 | 6 |
| Current spending | 2 | 3 | 0 | 5 |
| Future cuts in spending | 0 | 1 | 2 | 3 |

The Borda count now produces a clear winner! The choice has been influenced by the introduction of what might be called an 'irrelevant alternative.'

Decision rules that aren't 'independent of irrelevant alternatives' allow wily politicians to more easily manipulate the outcome of a decision making process to produce their most preferred outcome.

Rather than making persuasive arguments about the desirability of their most preferred outcome, a politician might get their way by the imaginative introduction of an alternative that has no chance of winning, but that can influence the alternative that's ultimately chosen.

Majority Rule with an Agenda Setter

Agenda Setting

An alternative decision-making mechanism that overcomes the potential instability of majority rule in round-robin tournaments requires actors to begin by considering only a subset of the available pair-wise alternatives.

A **voting agenda** is a plan that determines the sequence or order in which votes occur.

One possible voting agenda:

- First round: I vs. D .
- Second round: Winner of first round vs. C .

Pair-Wise Contests and Different Voting Agendas

| Agenda | 1st round | 1st-round winner | 2nd round | 2nd-round winner | Councillor obtaining their most preferred outcome |
|--------|-----------------------|------------------|-----------------------|------------------|---------------------------------------------------|
| 1 | <i>I</i> vs. <i>D</i> | <i>D</i> | <i>D</i> vs. <i>C</i> | <i>C</i> | Centrist councillor |
| 2 | <i>C</i> vs. <i>I</i> | <i>I</i> | <i>I</i> vs. <i>D</i> | <i>D</i> | Right-wing councillor |
| 3 | <i>C</i> vs. <i>D</i> | <i>C</i> | <i>C</i> vs. <i>I</i> | <i>I</i> | Left-wing councillor |

If everyone votes sincerely, the agenda setter can get their most preferred outcome. **The agenda setter is a dictator!**

But should we expect all the councillors to vote sincerely?

A **sincere vote** is a vote for an individual's most preferred option.

A **strategic or sophisticated vote** is a vote in which an individual votes in favor of a less preferred option because they believe doing so will ultimately produce a more preferred outcome.

Agenda 1: I vs. D , with winner against C .

The councillors know the second round will involve either D vs. C (C wins) or I vs. C (I wins).

Thus, the councillors know that if D wins the first round, the outcome will be C , and that if I wins the first round, the outcome will be I .

This means that the first round of voting is really a contest between C and I (even if they're voting on I and D).

Put yourself in the shoes of the right-wing councillor: $D > I > C$.

If they votes for their preferred option (D) in the first round, they'll end up with C (their worst preferred option) as the final outcome.

Thus, they have a strong incentive to vote strategically for I in the first round, since this will lead to I (their second preferred option) as the final outcome.

Some analysts find strategic voting lamentable and prefer decision rules that induce sincere voting.

We've seen that it's possible to avoid the potential for group intransitivity by imposing a voting agenda.

Unfortunately, the outcome of such a process is extremely sensitive to the agenda chosen, and, consequently, either of two things is likely to happen:

1. The instability of group decision making shifts from votes on outcomes to votes on the agendas expected to produce those outcomes.
2. Some subset of actors is given power to control the agenda and, therefore, considerable influence over the outcome likely to be produced.

Power of the agenda setter:

Their ability to specify a particular sequence of votes over the available alternatives.

But also their ability to determine the set of available alternatives that can be voted on in the first place.

With elections and referenda, voters only get to vote for the alternatives that appear on the ballot.

But who decided that these would be the only choices? Why don't other choices appear on the ballot?

If the set of ballot alternatives is restricted and thus doesn't reflect the full range of voter preferences, **how can we ever be confident in claiming the outcome of some vote reflects the will of the majority?**

UK Brexit Referendum, 2016:

It's not possible to know whether the Brexit decision was an accurate reflection of voter preferences and hence the revelation of the 'will of the people' or whether it simply reflects the institutional choices of the 'agenda setters' who chose to hold a referendum and restrict the options to the two that appeared on the ballot.

Restrictions on Preferences: The Median Voter Theorem

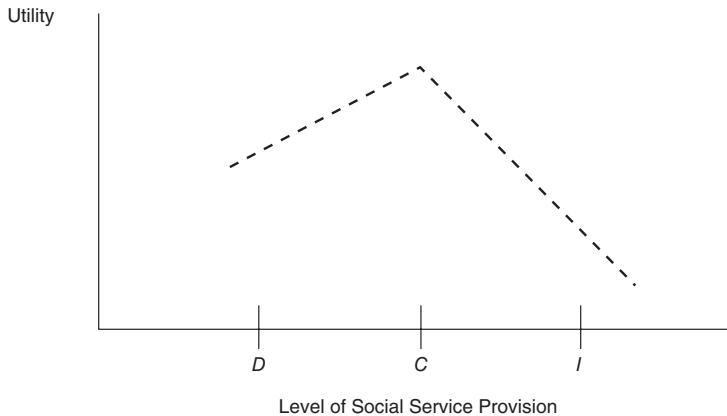
Another way in which stable outcomes might be produced is to restrict the preferences that actors can have.

It's possible to convey an individual's preference ordering in terms of a utility function.

A **utility function** is essentially a numerical scaling in which higher numbers stand for higher positions in an individual's preference ordering.

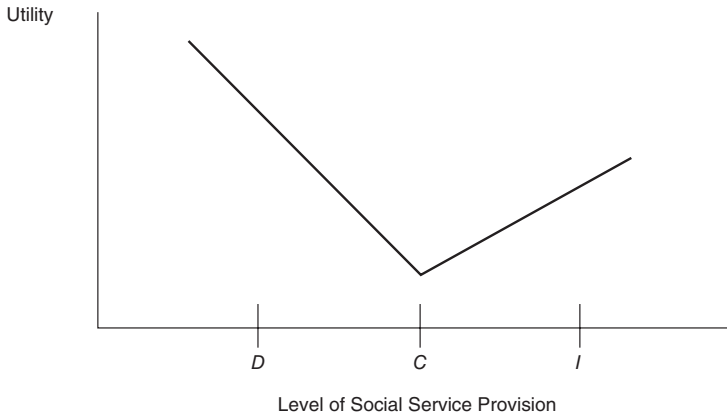
A **single-peaked preference ordering** is characterized by a utility function that reaches a maximum at some point and slopes away from this maximum on either side, such that a movement away from the maximum never raises the actor's utility.

Centrist Councillor's Utility Function



The centrist councillor has single-peaked preferences.

Right-Wing Councillor's Utility Function

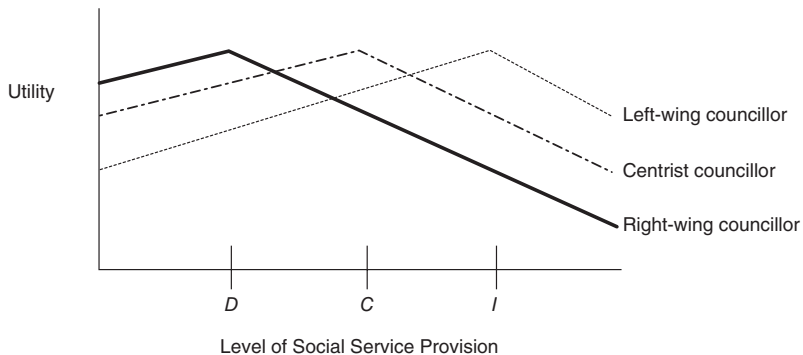


The right-wing councillor didn't have single-peaked preferences.

The **median voter theorem** states that the ideal point of the median voter will win against any alternative in a pair-wise majority-rule election if (1) the number of voters is odd, (2) voter preferences are single-peaked, (3) voter preferences are arrayed along a single-issue dimension, (4) and voters vote sincerely.

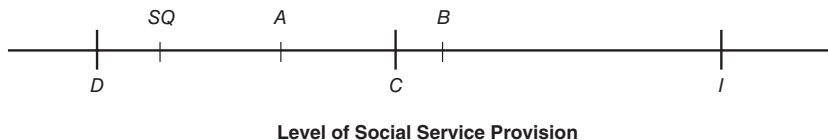
When voters are arrayed along a single-policy dimension in terms of their ideal points, the **median voter** is the individual who has at least half of all the voters at their position or to their right and at least half of all the voters at their position or to their left.

When All Three Councillors Have Single-Peaked Preference Orderings



C wins.

Illustrating the Power of the Median Voter



Any proposals will converge on the position of the median voter, C .

The MVT shows that the difficulties we encountered with Condorcet's Paradox can be avoided if we're willing to both rule certain preference orderings 'out of bounds' and reduce the policy space to a single dimension.

Unfortunately, neither of these restrictions is uncontroversial.

- There's nothing intrinsically troubling about individual preferences that aren't single-peaked.
- Many political questions are inherently multi-dimensional.

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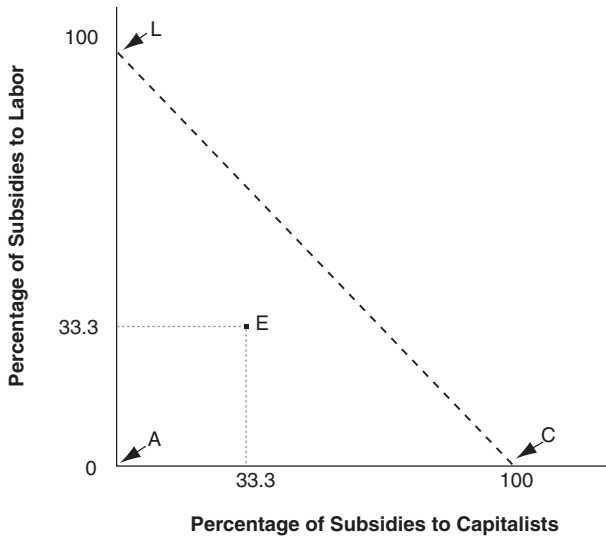
What if we increase the number of dimensions?

Labor, capital, and agriculture are deciding how to divide a pot of subsidies from the government's budget.

Each constituency only cares about maximizing subsidies to its own constituency.

The decision-making situation can be represented by a two-dimensional policy space.

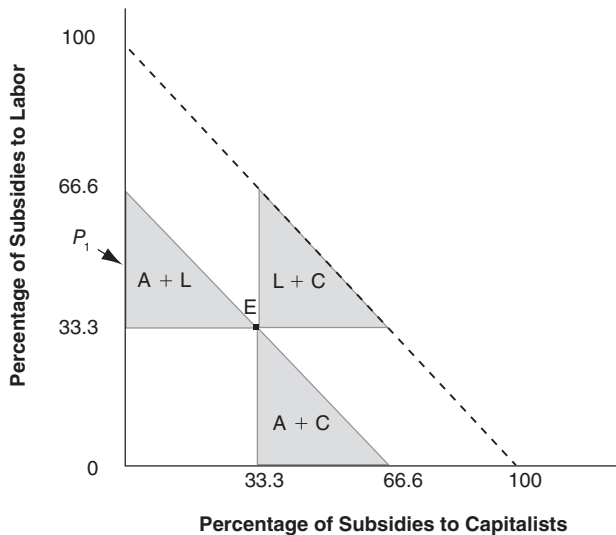
Two-Dimensional Voting



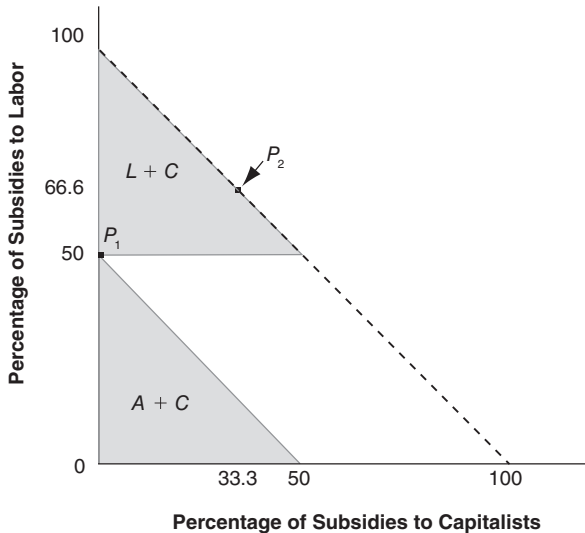
An **indifference curve** is a set of points such that an individual is indifferent between any two points in the set.

The **winset** of some alternative z is the set of alternatives that will defeat z in a pair-wise contest if everyone votes sincerely according to whatever voting rules are being used.

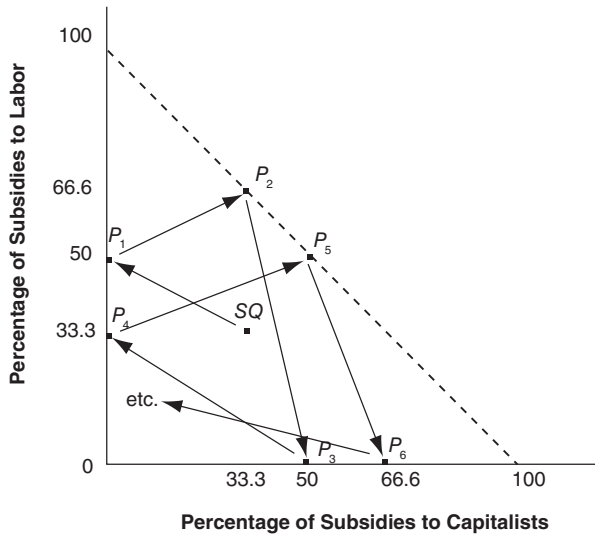
Two-Dimensional Voting with Winsets



Two-Dimensional Voting with a New Status Quo (P_1)



Two-Dimensional Voting with Cyclical Majorities



The **Chaos Theorem** states that if there are two or more issue dimensions and three or more voters with preferences in the issue space who all vote sincerely, then except in the case of a rare distribution of ideal points, there'll be no Condorcet winner.

Unless we're lucky enough to have a set of actors who hold preferences that don't lead to cyclical majorities, then either of two things will happen:

1. The decision-making process will be indeterminate and policy outcomes hopelessly unstable.
2. There'll exist an actor – the agenda setter – with the power to determine the order of votes in such a way that they can produce their most favored outcome.

Summary So Far

Condorcet's Paradox shows that a set of rational individuals can form a group that's incapable of choosing rationally in round-robin tournaments.

Alternative voting schemes like the Borda count allow clear winners in some cases, but the outcomes aren't necessarily robust.

If we employ 'single elimination' tournaments that form a voting agenda, the cyclical majorities may be avoided but whoever controls the agenda can dictate the outcome.

The problem of instability can be overcome if we have a single-issue dimension *and* each voter has single-peaked preferences.

But why should we restrict people's preferences and what about multi-dimensional problems?

So, should we just drop majority rule?

Arrow's Theorem

Arrow's Theorem states that it's impossible to design any decision-making procedure (not just majority rule) in which you rank alternatives that can guarantee producing a rational outcome while simultaneously meeting what he argued was a minimal standard of fairness.

Arrow presented **four fairness conditions** he believed all decision-making processes should meet.

1. The **non-dictatorship condition** states that there must be no individual who fully determines the outcome of the group decision-making process in disregard of the preferences of the other group members.

2. The **universal admissibility condition** states that individuals can adopt any rational preference ordering over the available alternatives.

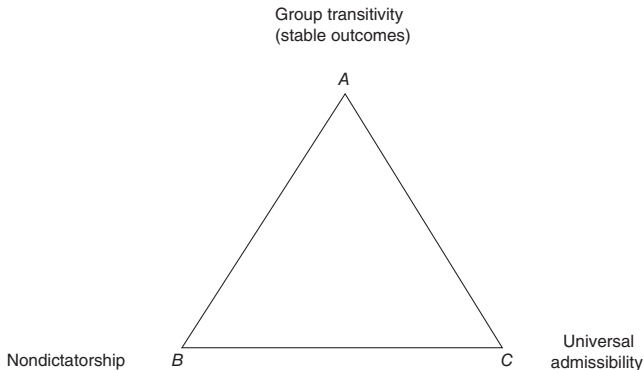
3. The **unanimity or pareto optimality condition** states that if all individuals in a group prefer x to y , then the group preferences must reflect a preference for x to y as well.

- Basically, the unanimity condition states that if everybody prefers x to y , the group shouldn't choose y if x is available.

4. The **independence of irrelevant alternatives (IIA) condition** states that group choice should be unperturbed by changes in the rankings of irrelevant alternatives.

- Suppose that, when confronted with a choice between x , y , and z , a group prefers x to y .
- The IIA condition states that if one individual alters their ranking of z , then the group must still prefer x to y .

Arrow's Institutional Trilemma



If we take Arrow's conditions of unanimity and IIA as uncontroversial, then we face an institutional 'trilemma' between stable outcomes, universal admissibility, and non-dictatorship.

Arrow's Theorem basically states that when designing institutions, we can choose one and only one side of the triangle.

- If we want group rationality and stable outcomes, we must give up either non-dictatorship or universal admissibility.
- If we want to avoid dictatorship, we must give up group rationality or universal admissibility.
- If we hold individual preferences inviolable, we must give up non-dictatorship or group rationality.

Arrow's Theorem shows that it's difficult to interpret the outcome of any group decision-making process as necessarily reflecting the will of the group.

When a group comes to a clear decision, it may mean individual preferences lined up in a way that allowed for a clear outcome that represented the desires of a large portion of the group.

But it may also mean that individuals with inconvenient preferences were excluded from the process or that some actor exercised agenda control.

In such cases, outcomes may reflect the interest of some powerful subset of the group rather than the preferences of the group as a whole, or even some majority of the group.

Every decision-making mechanism must grapple with the trade-offs posed by Arrow's Theorem, and every system of government represents a collection of such decision-making mechanisms.

Thus, we can evaluate different systems of government in terms of how their decision-making mechanisms tend to resolve the trade-offs between group rationality and Arrow's fairness criteria.

There is no perfect set of decision-making institutions.

Democracy is necessarily imperfect – Either fairness is compromised or there will be a potential for instability.

Legislative Intent?

A piece of legislation can't cover all conceivable contingencies for which it might be relevant.

This requires in any specific instance that a judge, bureaucrat, or lawyer must determine whether a specific statute is applicable.

Judges often ask, "What did Congress *intend* in passing this law?"

Liberals (in the American sense) have developed principles of statutory interpretation to enable broad meaning to be read into acts of Congress.

Conservatives, on the other hand, insist on requiring judges to stick to the plain meaning of the statutory language.

But who's right?

But who's right?

Short of appealing to our own prejudices and policy preferences, we can provide an analytical perspective based on Arrow's Theorem.

Arrow's Theorem cautions against assigning individual properties such as rationality to groups.

This makes it difficult, if not impossible, to talk about the **intent** of a group.

Individual **legislators** may have intentions. But this doesn't mean that we can talk about the intent of a **legislature**.

The Daily Show and Social Choice Theory [▶ here](#)